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ESTIMATING LOAD POWER PER UNIT VOLUME, DEVELOPED BY A PIEZOELECTRIC HIGH VOLTAGE GENERATOR

INTRODUCTION

The use of piezoelectric materials as generators (direct effect) of electrical energy has been generally confined to low signal devices. This has been due in part to mechanical limitations inherent in typical piezoelectric transducer materials. However, the development of the "spark pump" , made possible by the introduction of PZT-4 , has stimulated an interest in other high voltage applications for this and similar piezoelectric materials.

Typical devices that come to mind require sources of excitational energy which are generally easy to obtain. These include such things as hand operated levers, motor driven single or multiple acting cams, large acoustical or vibrational energies such as is present in the vicinity of heavy machinery and vehicles, etc. Much of this excitational energy can be considered wasted energy and the thought of effectively using it is indeed intriguing.

It should be noted, however, that piezoelectric transducers are characterized as high impedance devices (except, of course, when operated at extremely high frequencies). Consequently, the applications alluded to here, must be generally limited to low energy or power types.

However, as will be shown in the subsequent analyses, energy and hence power are functions of among other things, the volume of active material being utilized and, of course, power being the time derivative of energy also depends on the frequency of mechanical excitation. Thus where source conditions are appropriate and a sufficient volume of very active piezoelectric ceramics may be used, relatively high power is possible. It is recognized that this statement is only qualitative in nature. It now remains to show, in non rigorous fashion, the analysis which leads to quantitative information.

Before proceeding with the analysis, it becomes expedient to review some of the piezoelectric, dielectric, and elastic constants that will be encountered. These follow:

Piezoelectric Constants:

d_{hk} modulus relating strain to applied field or conversely relating charge density to applied stress,

g_{hk} modulus relating open circuit field to applied stress or conversely relating strain to applied charge density,

k_{hk} electromechanical coupling coefficient - a measure of the energy conversion capability.

Dielectric Constants:

$K_h^T = \epsilon_h/\epsilon_o$, relative dielectric constant (unclamped),

ϵ_h = permittivity (absolute dielectric constant) of material,

$\epsilon_o = 8.85 \times 10^{-12}$ f/m permittivity of free space.

Elastic Constant:

Y_{kk} = Young's modulus.

Explanation of subscripts:

The subscripts 1, 2, and 3 indicate the x, y, and z axes, respectively. For the piezoelectric constants, the first subscript refers to the direction of the field; the second subscript refers to the direction of strain. For the dielectric constant, the subscript notation is simplified and refers only to the direction of the field. For the elastic constant, the first subscript refers to the direction f stress, and the sec nd refers to the direction of strain.

Other terms to be encountered include:

ℓ = length of piezoelectric element

w = width of piezoelectric element

t = thickness of piezoelectric element

ω = $2\pi f$ = operating frequency expressed in radians

f = frequency of operation

volume = ℓwt

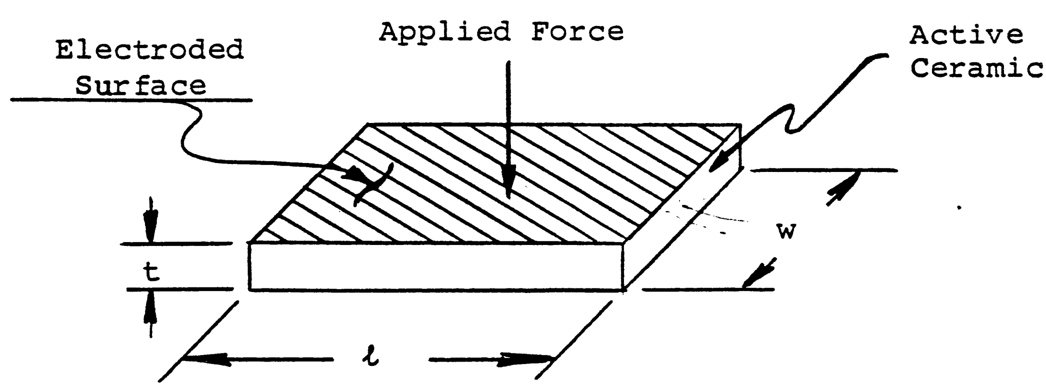
T = stress

S = strain.

ANALYSIS

Longitudinal Mode

The analysis t follow is based n a thickness expander plate. The stress is applied to the electroded faces as shown below.



The energy of a charged dielectric plate is given by equation (1):

$$En = \frac{1}{2} CV^2 \tag{1}$$

C = capacity of plate
V = voltage between electrodes.

V is, ultimately, the open circuit output voltage. Although its actual value is relatively unimportant at this point in the argument, it should be kept in mind that it is this voltage which eventually determines the thickness of the plate. Similarly, the capacity of the plate is also a function of the plate thickness and hence will also influence its overall height.

For the plate in question:

$$C = K_3 \epsilon_0 \frac{\ell w}{t} \tag{2}$$

From the definition of the g constant, we find

$$\frac{V}{t} = g_{33} \frac{F}{\ell w} = g_{33} T$$

F = applied force.

In order to determine the maximum available energy, we shall assume that T is the maximum available applied stress. (Note: For PZT-4 the maximum compressive stress should not exceed 15,000 psi for static operation and 7,000 psi for sustained cyclic operation.) Thus, since

$$V = g_{33} Tt, \tag{3}$$

substituting (2) and (3) into (1)

$$En = \frac{1}{2} K\epsilon_0 \frac{\ell w}{t} g_{33}{}^2 T^2 t^2$$

$$En = \frac{1}{2} K\epsilon_0 g_{33}{}^2 T^2 \ \ell wt. \qquad d_{33} = K \epsilon_0 g_{33}$$

$$= \frac{1}{2} d_{33}\ g_{33}\ T^2 \times \text{volume}$$

for PZT-4 = $3.5 \times 10^{-12}\ T^2$ joules/M^3 or 1.8×10^{-5} joules/M^3/psi

As indicated above, the time derivative of the energy developed is equal to the instantaneous power generated. Hence, it can be shown that for sinusoidal mechanical excitation, the maximum average power delivered to a resistive load is

$$P_{ave} = \frac{\omega}{2} \frac{K\epsilon_0}{2} g_{33}^2 T^2 \ell wt$$

if we assume a magnitude match or

$$R_\ell = \frac{1}{\omega C} .$$

It is also important to note that T is the maximum applied stress. Finally, the maximum average power out per unit volume of ceramic is

$$\frac{P_{ave}}{\ell wt} = \frac{\omega}{2} \frac{K\epsilon_0}{2} g_{33}^2 T^2 .$$

This is more conveniently written,

$$\frac{\text{Power Out}}{\text{Unit Volume}} = \frac{\omega}{4} \frac{k_{33}^2}{Y_{33}} T^2 . \qquad \frac{k_{33}^2}{Y_{33}} = K\epsilon_0 g_{33}^2$$

For PZT-4:

$$\frac{\text{Power Out}}{\text{Unit Volume}} = 1.85 \times 10^{-12} \omega T^2 \frac{\text{watts}}{\text{cu. meter}}$$

<u>Note</u>: The power referred to above is the average power. The rms power per unit volume in the load is

$$\frac{\text{Power}}{\text{Unit Volume}} = .463 \times 10^{-12} \omega T^2 \frac{\text{watts}}{\text{cu. meter}}$$

<u>Transverse Mode</u>

The assembly shown above can be driven in a direction perpendicular to the signal field. Although the coupling is lower (for example k_{31} of PZT-4 = 33%), this mode is more compliant than the thickness mode and therefore, the high volume efficiency may willingly be sacrificed in favor of a more compliant system.

For lateral excitation, equations (1) and (2) still apply. The voltage generated, however, is

$$V = g_{31} T_1 t \tag{4}$$

$$\text{where } T = \frac{F}{wt} .$$

Again substituting (4) and (2) into (1):

$$En = \frac{1}{2} K\epsilon_0 g_{31}{}^2 T_1{}^2 t^2 \frac{\ell w}{t} .$$

The average power delivered to the load is,

$$P_{ave} = \frac{\omega}{4} K\epsilon_0 g_{31}{}^2 \ell wt \ T^2 .$$

Finally, the average power out per unit volume of ceramic is,

$$\frac{P_{ave}}{\text{Volume}} = \frac{\omega}{4} K\epsilon_0 g_{31}{}^2 T^2 .$$

In terms of coupling, this can be written

$$\frac{P_{ave}}{\text{Unit Volume}} = \frac{\omega}{4} \frac{k_{31}{}^2}{Y_{11}} T^2 .$$

$$= .412 \times 10^{-12} \omega T^2 \frac{\text{watts}}{\text{cu. meter}}$$

Once again, the rms power per unit volume is

$$\frac{\text{Power}}{\text{Unit Volume}} = .103 \times 10^{-12} \omega T^2 \frac{\text{watts}}{\text{cu. meter}}$$

It is seen that for conditions of applied stress level frequency similar to those outlined for the thickness mode above, approximately 5 times the ceramic is required to maintain the same output power level.

Other modes of operation are conceivable. Only the two most effective modes of operation have been discussed above. However, these other modes of operation should only be considered if the minimum volume of material as determined from the expressions above is well below the maximum allowable volume.