THEORETICAL POWER LIMITS OF SONAR TRANSDUCERS

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Summary. The power output of a sonar transducer may be considered to have an acoustical, an electrical, a mechanical, and a thermal limit. Many transducers encounter neither the acoustical, or cavitation, limit nor the thermal limit because of favorable operating conditions. The electrical and mechanical limits, on the other hand, are of practical importance in all high performance transducers. These limits are expressible in terms of fundamental transducer parameters, such as electromechanical coupling coefficient, mechanical Q, and maximum storable electric and elastic energy densities. The usefulness of various criteria for comparing power capability, such as watts/cm^2 and watts/lb, is discussed.

Introduction

The power output of a sonar transducer may be considered to have an acoustical, an electrical, a mechanical, and a thermal limit. The acoustical limit is reached when pronounced cavitation sets in, and it is determined by three factors: the compressive bias on the water due to depth, the dynamic tensile strength of the sea water, and the near-field pressure distribution of the radiated sound. While the cavitation limit is of prime importance for surface vessel sonars, many other sonars operate at depths sufficiently great so that this limit is not encountered.

The thermal limit is a function of the efficiency, duty cycle, and thermal conductivity designed into the transducer. Few generalizations about thermal design may be made, since transducers come in such a variety of configurations. In many sonars the transmitted signal has such a low duty cycle that the thermal limit is not encountered.

The power limits which will be considered in detail here are the electrical and the mechanical limits, since these are of practical importance in all high-performance transducers. The analysis will be for the common single-degree-of-freedom transducer, and we shall formulate equations for the electrical and mechanical power limits in terms of basic transducer parameters such as electromechanical coupling coefficient, mechanical Q, and the storable electric and elastic energies.

The Electrical Limit

The electrical limit will be treated first. Figure 1 shows the equivalent circuits which will be employed and shows relations of some of the basic parameters to circuit element values. The electromechanical coupling coefficient k is a function of the motional capacity C_Y and the blocked capacity C_b for the electric field transducer, or the motional inductance L_z and the blocked inductance L_b for the magnetic field transducer. The mechanical resonant frequency ω_r is determined by the motional inductance and capacity. The mechanical storage factor at resonance, Q_M, is a function of the motional capacity or inductance and the motional resistances. The power absorbed in the motional resistances R_Y and R_Z is partly useful radiated power and partly internally dissipated power.

In Fig. 2 expressions are given for the electric and magnetic energies stored in the blocked capacity or blocked inductance of the transducer. For most transducers the electric and magnetic fields are uniform throughout the active material and the second form of these equations then apply, in which ϵ_b is the blocked permittivity, $\mathcal{E}_p$ is the peak value of the alternating electric field, μ_b is the blocked permeability, H_p is the peak value of the alternating magnetic field, and V is the volume of material in which the electric or magnetic energy is stored. The next equations give the radiated power; the mechanoacoustical efficiency η_{ma} has been introduced to obtain this radiated portion from the total mechanical power flowing into the motional resistances. In these equations we shall eliminate the voltage E and current I in favor of the stored energies given by the equations above and shall next eliminate the various circuit elements by use of equations given in Fig. 1. The result is the general equation at the bottom of Fig. 2, which applies to all types of linear electroacoustical transducers.

The transducer's radiated power is limited because the storage capacity of its mechanically coupled electric or magnetic reservoir is limited. The limits on the stored energy U_e are imposed by such factors as insulation breakdown, depolarization of materials operating at remanence, distortion resulting from dielectric or ferromagnetic nonlinearities, and deterioration of efficiency resulting from a rise in the dissipation factor at high fields. Nonlinearities are thus included as a possible power limit, although the power equation was derived on the basis of linear circuit theory. Its usefulness, therefore, is restricted by the assumption that grossly nonlinear operation will be avoided.

The Mechanical Limit

Next, the mechanical limit, which depends on the transducer's capacity to store elastic energy, will be derived. The starting point is the power equation at the top of Fig. 3. By expressing the radiation resistance as a function of Q_M and eliminating the velocity in favor of the elastic energy, we obtain the general equation at the center of Fig. 3, which is free from explicit dependence on the equivalent circuit elements. The limit on the elastic energy U_m, which in turn limits the radiated power, is caused by factors such as fracture in the case of ceramic or crystal transducers, metal fatigue for transducers employing springs, diaphragms, or magnetostrictive laminations, gap closure for variable reluctance or electrostatic transducers, and excessive travel for moving coil transducers.

The transducer power must remain below whichever of the two limits included in Figs. 2 and 3 is lower. For an economical design, it is desirable to have the electrical and the mechanical power limits equal. When the optimum radiation loading to bring about this condition prevails, the mechanical Q will have the value given by the equation at the bottom of Fig. 3.

Maximum Storable Energy Densities

The two power-limit expressions which have been presented depend on the resonant frequency and the transducer size, inasmuch as the storable energies are a function of size. We shall now introduce some different parameters, defined in Fig. 4, which will make the results independent of resonant frequency and less dependent on transducer size. Energy densities rather than total energies will be used, and the radiated power will be divided by the radiating area to yield an average surface intensity. Other parameters required are the radiation resistance per unit area (averaged), the maximum value of the stress, T_{max}, which can be allowed without risking fracture or fatigue, and the density ρ and sound velocity c of the active material.

Before returning to the power limit equations we shall consider the energy densities which seem feasible for the common piezoelectric and piezomagnetic transducer materials (see Fig. 5). The first column lists the electromechanical coupling coefficient which is available when the material is used in its most favorable mode. The next column gives the estimated maximum electric or magnetic energy density. The third column gives the product of two of the factors which occur in the electrically limited power equation; it indicates the amount of electrical energy which has been converted to mechanical energy and is available for work. The last column pertains to mechanically limited conditions and gives the estimated maximum elastic energy density. All these estimates are extremely crude, and they are offered as targets for criticism rather than as definitive limits.

The Ring, Flexural Disk, and Tonpilz Transducers

Next, the two power-limit equations will be applied to three different types of transducers, but the results will be expressed as surface intensity rather than total acoustic power. In all cases radiation mass will be neglected. We consider first the piezoelectric or piezomagnetic ring resonator (see Fig. 6).

In its radial mode the resonant ring has uniform stress distribution as long as it is reasonably thin, and hence the elastic energy equation is simple. The ring can be constructed so as to utilize the highest coupling coefficient of the active material. The intensity limits are independent of resonant frequency, and size does not appear explicitly in their equations, though it must be such as to maintain the resonant condition. The mechanical Q equation assumes radiation from one cylindrical surface only, and it depends on the thickness-to-radius ratio b/a of the ring. The specific radiation resistance r_r is, of course, a function of the height of the ring, but in an array of transducers it is also a function of the spacing between transducers and hence is subject to considerable control.

Figure 7 gives similar equations for the edge-supported flexural disk transducer. The flexural disk is made of laminated ceramic, with the laminations on opposite sides of the neutral surface driven with opposite polarities so that flexural stresses are generated piezoelectrically. The intensity equations have the same form as that for the ring except for numerical factors, which account for the more complex velocity and stress distributions occurring in flexural vibrations. The mechanical Q for the disk depends on the square of the thickness-to-radius ratio, and the coupling coefficient is a function of the planar coupling coefficient of the material.

Figure 8 gives the equations for a tonpilz, that is, a transducer consisting of a piezoelectric or piezomagnetic elastic member loaded on each end with masses. The transducer is assumed to radiate only from Mass 1. The tonpilz designer has a number of dimensional ratios at his disposal; accordingly, the equations must depend on these design variables as well as on the factors which were common to the equations for the two transducer types presented previously.

Intensity Limits of Barium Titanate Transducers

Next, the intensity limits for the three transducer types discussed above will be calculated in terms of the properties of a particular material. Barium titanate is chosen as the active material for all three transducer types; the maximum allowable stress is taken to be 3000 psi and the maximum electric driving field to be 2 KV/cm rms. For this illustrative example the specific radiation resistance r_r is assumed to remain constant at 1/2 the ρc of water, and the mechanoacoustical efficiency is given a constant value of 60%.

In Fig. 9 the intensity limits are plotted as a function of mechanical Q. The Q is varied by varying a dimensional ratio of the transducer over what seems like a practical range. The thickness-to-radius ratio is varied for the ring and the flexural disk transducers, and the ratio of active material area to radiating area is varied for the tonpilz transducer. In order to fix the design of the tonpilz it was necessary to choose values for the other two dimensional ratios appearing in the equations given in Fig. 8; the ratio of radiating mass to non-radiating mass was chosen to be 1 to 3, and the ratio of front mass length to active material length was chosen as 1 to 4. The condition that the specific radiation resistance remain constant as the dimensional ratios of the transducers are varied might be brought about reasonably well in an array by simultaneous variation of the spacing between transducers.

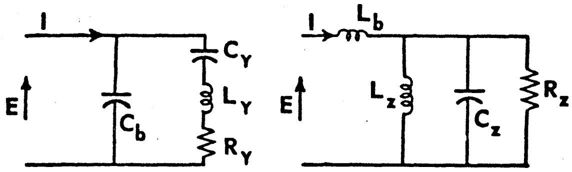
The results given in Fig. 9 are, of course, independent of the frequency of the design, though they do assume operation at resonance. Sonars normally operate in the lower decade of intensities covered on this graph. It is seen that the ring and the tonpilz appear capable of intensities extending into the second decade, while the flexural disk output falls near the bottom of the lower decade.

Watts/cm^2 versus Watts/lb as Criteria

Watts/cm^2 values, such as displayed here, are a useful indication of a transducer's capability, but they do not serve as a criterion of design excellence when different transducer types are compared. Thus, to evaluate these examples it would be necessary to give consideration to the amount of material in back of the radiating aperture required to produce the intensities shown. Since weight is usually of major importance in high-power sonar systems, watts/lb is a figure of interest. When a given design is scaled in frequency, its watts/lb figure will be directly proportional to frequency. If we choose a frequency of 5 kc and compute the watts/lb figures of the three transducer types at the value of mechanical Q where the electrical and mechanical limits are equal (or at the lower end of the plotted curves for the tonpilz), we obtain the following results: the ring ranks highest at 800 watts/lb, the flexural disk is next at 130 watts/lb, and the tonpilz is lowest at 60 watts/lb.

In conclusion, the examples which have been given are particular cases of a rather general analysis of power limits. The general method applies to variable reluctance, electrostatic, moving coil, and piezomagnetic transducers, as well as to the piezoelectric types illustrated.

ELECTRIC FIELD TRANSDUCER | **MAGNETIC FIELD TRANSDUCER**



Electric field transducer:

$$\omega_r^2 = \frac{1}{L_Y C_Y}$$

$$Q_M = \frac{1}{\omega_r C_Y R_Y}$$

$$\frac{k^2}{1-k^2} = \frac{C_Y}{C_b}$$

Magnetic field transducer:

$$\omega_r^2 = \frac{1}{C_z L_z}$$

$$Q_M = \frac{R_z}{\omega_r L_z}$$

$$\frac{k^2}{1-k^2} = \frac{L_z}{L_b}$$

Fig. 1. Transducer equivalent circuits.

ELECTRIC FIELD | **MAGNETIC FIELD**

PEAK STORED ENERGY, TRANSDUCER BLOCKED:

$$U_e = \frac{1}{2}|E|^2 C_b = \frac{1}{2}\epsilon_b \mathcal{E}_p^2 V \qquad U_e = \frac{1}{2}|I|^2 L_b = \frac{1}{2}\mu_b H_p^2 V$$

RADIATED POWER AT RESONANCE:

$$P_r = \left(\frac{1}{2}|E|^2/R_Y\right)\eta_{ma} \qquad P_r = \frac{1}{2}|I|^2 R_z \eta_{ma}$$

$$\boxed{P_r = \eta_{ma}\,\omega_r \frac{k^2}{1-k^2} Q_M U_e}$$

η_{ma} = MECHANOACOUSTICAL EFFICIENCY

V = VOLUME OF ACTIVE MATERIAL

Fig. 2. The electrical limit equation.

RADIATED POWER: $P_r = \frac{1}{2}|v|^2 R_r$

PEAK ELASTIC ENERGY: $U_m = \frac{1}{2}\frac{|v|^2}{\omega^2 C_M}$

RADIATION RESISTANCE: $R_r = \frac{\eta_{ma}}{\omega_r C_M Q_M}$

v = PEAK VELOCITY

C_M = TRANSDUCER COMPLIANCE

$$\boxed{P_r = \frac{\eta_{ma}\omega_r U_m}{Q_M}}$$

OPTIMUM Q_M FOR POWER: $Q_M = \sqrt{\dfrac{(U_m)_{max}}{\frac{k^2}{1-k^2}(U_e)_{max}}}$

Fig. 3. The mechanical limit equation.

DEFINITIONS

$(u_e)_{max} = (U_e)_{max}/V$ = MAXIMUM ELECTRIC (OR MAGNETIC) ENERGY DENSITY.

$(u_m)_{max} = (U_m)_{max}/V$ = MAXIMUM ELASTIC ENERGY, AVERAGED OVER THE VOLUME.

T_{max} = MAXIMUM ALLOWABLE STRESS.

$I_s = P_r/A_r$ = SURFACE INTENSITY OF RADIATION.

$r_r = R_r/A_r$ = SPECIFIC RADIATION RESISTANCE.

A_r = RADIATING AREA.

ρ = DENSITY OF ACTIVE MATERIAL.

c = SOUND VELOCITY OF ACTIVE MATERIAL.

Fig. 4. Additional definitions.

MATERIALS	COUP. COEFF. $k = k_{33}$	ESTIMATED ENERGIES IN JOULES/m^3: $(u_e)_{max}$	$\frac{k^2}{1-k^2}(u_e)_{max}$	$T_{max}^2/2\rho c^2$ $(u_m)_{max}$
NICKEL	.30	200	20	4000
PERMENDUR POLARIZED	.29	400	37	4000
PERMENDUR REMANENCE	.20	90	4	4000
NICKEL FERRITE	.32	90	10	1500
ADP CRYSTAL	.28	200	17	4500
BARIUM TITANATE	.48	400	120	2000
LEAD TITANATE ZIRCONATE	.60	1200	670	3000

Fig. 5. Energy limits of transducer materials.

RING TRANSDUCER

a = MEAN RADIUS b = RING THICKNESS

MAX. ELASTIC ENERGY: $U_m = \frac{T_{max}^2}{2\rho c^2} V$

ELEC. LIMIT: $I_s = Q_M \frac{k^2}{1-k^2}\frac{r_r}{\rho}(u_e)_{max}$

MECH. LIMIT: $I_s = \frac{r_r}{2\rho^2 c^2} T_{max}^2$

STORAGE FACTOR: $Q_M = \frac{b}{a}\frac{\rho c}{r_r}\eta_{ma}$

MAX. COUPLING COEFF.: $k = k_{33}$

Fig. 6.

SUPPORTED-EDGE FLEXURAL DISK

a = DISK RADIUS $\quad h$ = DISK THICKNESS

MAX. ELASTIC ENERGY: $U_m = .15 \dfrac{T_{max}^2}{2\rho c^2} V$

ELEC. LIMIT: $I_s = .71 \, Q_M^2 \dfrac{k^2}{1-k^2} \dfrac{r_r}{\rho} (u_e)_{max}$

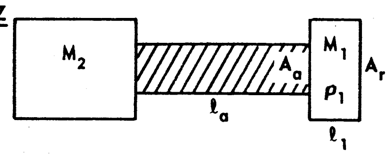
MECH. LIMIT: $I_s = .10 \dfrac{r_r}{2\rho^2 c^2} T_{max}^2$

STORAGE FACTOR: $Q_M = 2.0 \left(\dfrac{h}{a}\right)^2 \dfrac{\rho c}{r_r} \eta_{ma}$

COUPLING COEFF.: $k = .77 k_p \approx .5 k_{33}$

Fig. 7.

TONPILZ



$M_T = M_1 + M_2$, $V = A_a \ell_a$ $\quad U_m = \dfrac{T_{max}^2}{2\rho c^2} V$

MAXIMUM $k \leq k_{33}$

ELEC. LIMIT: $I_s = \left(\dfrac{M_2}{M_T}\right)\left(\dfrac{\ell_a}{\ell_1}\right)\left(\dfrac{A_a}{A_r}\right) Q_M^2 \dfrac{k^2}{1-k^2} \dfrac{r_r}{\rho} (u_e)_{max}$

MECH. LIMIT: $I_s = \left(\dfrac{M_2}{M_T}\right)\left(\dfrac{\ell_a}{\ell_1}\right)\left(\dfrac{A_a}{A_r}\right) \dfrac{r_r}{2\rho^2 c^2} T_{max}^2$

STORAGE FACTOR: $Q_M = \left(\dfrac{M_T}{M_2}\right)^{3/2} \sqrt{\dfrac{\ell_1 A_a}{\ell_a A_r}} \dfrac{\sqrt{\rho \rho_1} c}{r_r} \eta_{ma}$

Fig. 8. Tonpilz transducer.

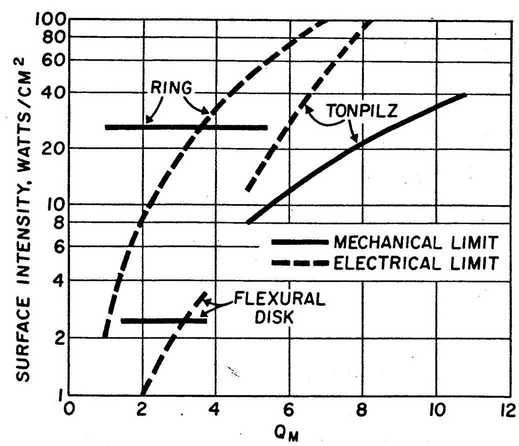


Fig. 9. Intensity limits of barium titanate transducers.