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A CRITICAL REVIEW OF THE CRITERIA FOR NOTCH-SENSITIVITY IN FATIGUE OF METALS

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A CRITICAL REVIEW OF THE CRITERIA FOR NOTCH-SENSITIVITY IN FATIGUE OF METALS

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ABSTRACT

It is the purpose of this bulletin to summarize and to appraise critically the numerous interpretations or correlating methods that have been proposed in the technical literature to compare the endurance limits of notched rotating beam fatigue specimens with those of unnotched specimens. The interrelation of the ideas proposed by several investigators was studied. The discrepancy between theoretical and effective stress concentration factors is attributed to the fact that the structural action in real materials is different from that of a homogeneous, isotropic, elastic, "idealized material" commonly assumed in the theoretical analysis of stresses.

After surveying all relevant hypotheses it was summarized that notch-sensitivity of a metal member depends upon three different factors, namely: (a) the basic material characteristics, of which the localized work-hardening (or strain strengthening) capacity may be considered as an index; (b) the degree of material homogeneity, which is influenced by inherent defects, tensile residual stresses, heat treatments, etc.; (c) the geometry of the specimen (including over-all size), the radius at the root of the notch being of prime importance in this geometric factor. It was concluded that the criterion for fatigue failure or for endurance limit should include not only the peak stress at a critical *point* as is conventionally assumed, but also the conditions existing in a critical *region* surrounding the point. A rational approach and procedure for attacking the problem of notch effect as well as size effect is suggested.

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CONTENTS

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FIGURES

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I. INTRODUCTION

1. Importance of Notches

It has been well known[1]* for many years that most failures in machine parts (and in some structural members) are *progressive fractures* resulting from repeated load; these "fatigue failures" nearly always start at an imposed or accidental discontinuity, such as a notch or hole. For example[21], fatigue fracture developing from a small hole wrecked a huge costly turbine; a sharp fillet on an axle caused a serious accident in a school bus; stamped marks of inspection on a propeller resulted in the crash of an airplane. Thus a tiny "notch" is frequently a potential nucleus of fatigue failure which may lead to serious damage.

The term "notch" in a broad sense is used to refer to any discontinuity in shape or nonuniformity in material. A notch is frequently called a "stress raiser" because it develops localized stresses that may serve to initiate a fatigue crack (or reduce the load-carrying capacity). Notches are hardly avoidable in engineering practice; they may occur as (a) a metallurgical notch, which is inherent in the material due to metallurgical processes (as inclusions, blowholes, laminations, quenching cracks, etc.); (b) a mechanical notch, of some geometrical type which usually results from a machining process (as grooves, holes, threads, keyways, fillets, serrations, surface indentations); (c) a service notch, which is formed during use (as chemical or corrosion pits, scuffing, chafing or fretting, impact indentations, and so on). Hence the potential load-carrying capacity of a material under repeated stress can seldom be attained in actual machine parts because of the presence of these imposed or accidental notches. The fatigue "notch-sensitivity," or susceptibility of a member to succumb to the damaging effects of stress-raising notches (this susceptibility varies with different materials) is therefore an important consideration in almost every branch of machine design involving the proportioning of members for service under repeated stress.

Usually the term notch as used in its narrow sense refers only to notches of type (b) — i.e., the mechanical notch. The pronounced reduction of fatigue strength due to a sudden change in cross-section of a load-carrying member has been noted in many experimental investigations ever since the classical tests of Wöhler[3]. Many investigators have

* Parenthesized superscripts refer to correspondingly numbered entries in the Bibliography.

proposed numerous methods of interpreting the available fatigue test data obtained from notched specimens. The large number and variety of these hypotheses, interpretations, and correlating methods employed to compare the endurance limits for notched specimens with those of unnotched specimens have led to confusion and unsatisfactory results from the viewpoint of factual knowledge for the machine designer.

2. Purpose and Scope

It is the purpose of this report to review briefly each of these interpretations and to appraise them critically in terms of physical significance and of agreement with experimental test data. The interrelations of these different methods and the general basic concepts are analyzed. The probable fundamental factors involved in the "notch effect" and a rational procedure for attacking the problem of notch-sensitivity are suggested.

In general, most of these previous analyses were primarily qualitative and based only on a single concept of basic material behavior, such as plasticity, damping capacity, cohesive strength, work-hardening capacity, elementary structural unit, or statistical theories of fatigue. Several of these approaches are discussed in Chapters II and III. Interpretations based upon static tensile strength or impact strength are only indirect and accidental, and have not led to an accurate and functional correlation. Some attempts at rationalization have been based on the stress conditions as affected by the geometry of the notch, such as state of stress, shear energy theory, stress gradient and extent of stress concentration. These are discussed in Chapters IV and V. Other quantitative approaches, such as correlations based on notch radius, or of stress at a given depth below the surface (discussed in Chapter VI), represent a type of empirical approach deduced from speculative thinking which lacks evidence in the light of recent fatigue tests or theories. Other developments based on statistical effects and the homogeneity of metals are presented in Chapters VII and VIII.

However, these approaches all seem to indicate a partial truth regarding the behavior of a notched specimen. They are of value in recording various attempts at correlating data, and may guide our attempts to predict the notched fatigue strength of a member until the knowledge on this problem is further advanced.

3. Comparison of Theoretical and Effective Stress Concentration Factors

When a notch is introduced in a specimen subjected to an elastic static load, the stress at the root of the notch is markedly increased. The ratio of the value of the peak stress in the notched member to that in a corresponding unnotched member is called the "theoretical stress concentration factor," K_t*. The peak stress in the notched specimen may be determined either mathematically[4, 5], photoelastically[2, 6], or by X-ray measurement[7], while the peak stress in the unnotched specimen is always calculated from the elementary stress formulas (such as $S = P/A$, $S = Mc/I$, and $S = Tc/J$ for axial, bending, or torsional loads respectively[1]).

Since the peak stress is raised by the factor K_t, it might be expected that the strength of the notched specimen would be reduced by the factor K_t. Experimentally it is observed that the amount of reduction of load-carrying capacity due to a notch roughly tends to increase with (but always is smaller than) the factor K_t. The ratio of the endurance limit of an unnotched specimen to that for a notched specimen is called the "strength reduction factor" or "effective stress concentration factor," K_e. The endurance limits are determined by fatigue testing in which only elementary stress formulas are used for calculating the stresses in the specimens, whether notched or unnotched.

The discrepancy between theoretical and effective stress concentration factors K_t and K_e varies not only for different metals but also for different sizes of specimen and different types of notch; the lack of a rational explanation for these variations has led to much confusion and speculation. The fundamental cause for this discrepancy may be attributed to the fact that the response of a material subjected to a repeated loading is quite different from the behavior of the same material when subjected to an elastic static loading. The analyses upon which the theoretical factors are based depend on the assumptions of an isotropic material which is perfectly elastic and homogeneous and whose stress conditions and strength properties are not influenced by time or temperature. However, when dealing with fatigue tests of metals, the small localized spots (crystals, slip bands or grain boundaries) in which fatigue failure initiates are anisotropic and far from homogeneous. Localized inelastic readjustments which are sensitive to time and temperature and which alter the stress and strength occur in the material at stress levels as low as its endurance limit, or even lower. Better understanding of the mechanism of deformation in polycrystalline metals under repeated loading will therefore help to clarify the reason for the discrepancy between K_t and K_e.

When a metal piece is said to be notch-sensitive, it is inferred that the ratio K_e/K_t for that piece is relatively high; that is, the value of strength reduction factor K_e is relatively high with respect to the value of K_t. In most cases the value of K_e lies between 1 and K_t, but there are

* In many instances the value of K_t is defined as the ratio of the peak stress in the notched member to the nominal stress computed from the dimensions of the *minimum section* at the notch. For comparative purposes either definition is acceptable, but the values of K_t are slightly different.

occasional exceptions. For example, for some stainless steels the value of K_e may be less than one (Section 9) and for some quenched and tempered steels K_e it may sometimes be greater than K_t[8]. The value of K_e is directly proportional to the value of K_t *only* when the "notch-sensitivity" is constant.

4. Comparison of Notch-sensitivity in Fatigue and Static Tension

The problem of notch-sensitivity of metals has been investigated in experiments employing three different types of loading — static tension, Charpy impact, and rotating-beam fatigue tests. From results of static tension tests[9, 10, 11, 12] it has been shown that for ductile steels the ultimate strength and yield strength increase with notch depth and sharpness of the notch angle; the breaking stress (load per unit of *actual* area at fracture) also increases moderately or remains approximately constant. For brittle metals such as cast iron, cast brass, and magnesium alloys (which may have many internal defects or high residual stresses but little capacity for plastic flow) there often is little difference between the strength values of notched and unnotched bars.

For cast iron and some cast aluminum alloys under repeated loading there is also little difference between the values of the endurance limit for notched and unnotched bars[13]. For ductile metals, however, the presence of a notch usually reduces the fatigue strength. Under repeated loading a ductile material does not undergo large-scale plastic flow at the notch root or large extension and rounding out of the notch, both of which tend to relieve the stress concentration. Experimentally it is generally found that soft steels (which have higher tensile strength in notched specimens than in unnotched ones) are not highly notch-sensitive in fatigue tests; on the other hand, hard steels which show reduced *static* tensile strength due to a sharp notch are also very notch-sensitive in fatigue. These observations of the behavior of soft and hard steels resulted in one attempt at a correlation of fatigue notch-strength with static tensile strength, as is discussed in Section 8.

5. Comparison of Fatigue and Impact Notch-sensitivity

It is generally believed that hard steels are more notch-sensitive than soft steels either in a fatigue test, a static tension test, or an impact test. Some test data have indicated that the stronger the steel the lower is the Charpy impact value[13] and the greater the fatigue notch-sensitivity; hence, one might infer the possibility of a relation between *impact* values and *fatigue* notch-sensitivity. However, no direct correlation between these two types of test has ever been reported[14, 21] and some contrary evidence indicating that there is no reason to expect a correlation has been presented[8].

In a study by Dolan and Yen[8], experimental data were presented from fatigue and Charpy tests on two alloy steels and one carbon steel, heat-treated in several different ways to approximately the same hardness level. It was concluded that no direct functional relationship was evident between the concepts of notch-sensitivity in a fatigue test and the notch-sensitivity evidenced in a Charpy impact test. A rough qualitative correlation was indicated for comparisons of the same material at the same hardness and tensile strength level; but even this qualitative relationship was inaccurate (and the relative order of notch-sensitivity was reversed in the two types of test) when comparing materials of different chemical analyses.

The Charpy test develops a higher rate of strain at peak stress than the fatigue test. Fracture under a single impact is not dependent upon the cumulative chance effects developed during the repetitions and reversal of load which are of paramount importance in the submicroscopic phenomena leading to failure in fatigue. Therefore, direct correlations between fatigue properties and Charpy values do not seem feasible.

II. INTERPRETATIONS BASED ON CONCEPTS OF MATERIAL BEHAVIOR

6. Plasticity

Moore in 1931 reported[15] a new localized "plasticity" property of metals, that is, the ability to stand occasional overstress in localized zones without developing a crack. He called this "crackless plasticity" and described it as a property unique to the conditions encountered in repeated loading, and one which could not be measured by the ductility in static tests. For example, the ductility of an alloy steel was much higher than that of hard spring steel, but neither steel showed a high degree of crackless plasticity (or notch-insensitivity) under repeated stress. Copper-nickel alloys exhibited good elongation and reduction of area in a static test, but in fatigue tests did not resist localized plastic deformation without starting a crack. Pure metals and very fine-grained metals appeared most sensitive to the effect of notches.

Thum in 1932[16] and several others[49] claimed that the theoretical peak stress is lowered by plastic action which redistributes the stress and decreases the effective stress concentration factor. It was implied that differences in notch-sensitivity of various metals were due to the relative degrees of lowering of the peak stress. If S_n denotes the nominal stress in a notched specimen as found by the elementary formula, then K_tS_n will be the value of the theoretical peak stress and K_eS_n will be considered as that of the "actual" peak stress as shown in Fig. 1. The ratio of the increase of the "actual" peak stress, K_eS_n, over the nominal stress S_n, to the increase of the theoretical peak stress, K_tS_n, over the nominal stress S_n, was regarded as a material property. It has been called the "notch-sensitivity index," q. That is:

$$q = \frac{K_eS_n - S_n}{K_tS_n - S_n} = \frac{K_e - 1}{K_t - 1} \tag{1}$$

whence K_e may be found if q for a material and K_t for any notch are known:

$$K_e = 1 + q\,(K_t - 1) \tag{1a}$$

The values of q vary between 0 and 1 as the values of K_e vary between 1 and K_t, but test values are occasionally found beyond these limits, as has been discussed in Section 3. The greater the value of q, the greater the notch-sensitivity of the material.

Peterson[17] showed that the notch-sensitivity index q depended not only on the properties of the material itself, but also on the shape and dimensions of the test piece. This peculiarity complicates the estimation of fatigue strength of notched members, and indicates that q is *not* a fundamental "material property."

Föppl[18] suggested that damping capacity, a property of the material independent of the dimensions of the test piece, was a measure of plastic strain and hence was likewise a measure of notch-sensitivity. Several other investigators[49] also tried to correlate damping capacity with notch-sensitivity on the basis of the observation that some metals of low notch-sensitivity, such as cast iron, possess high damping capacity. However, there are exceptions to this statement; and since the complicated phenomena and structural actions involved in either fatigue or damping are not fully understood, any direct quantitative correlation does not at present appear feasible[49].

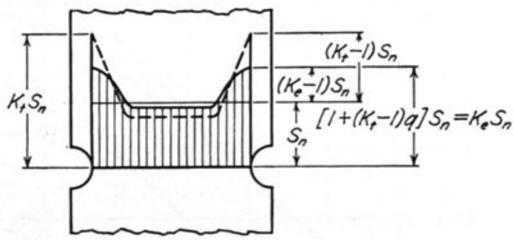

Fig. 1. Lowering of Peak Stress by Plastic Action (from reference 16)

It has been a general concept that plastic action mitigates peak stress in fatigue loading. However, as discussed by Yen[50], the strain-hardening accompanying repeated loading gradually reduces the subsequent plastic deformation; this tends to make the *range* of peak stress in each cycle approach the value calculated by elastic theory. This may be related to the observation that no lowering of stress at notches was observed by X-ray measurements under alternating stresses not exceeding the yield limit[20, 49]. Consequently the hypothesis that lowering of the range of peak stress is due to plasticity is not confirmed either by theoretical study or by experiment.

Since the fatigue phenomena are initiated on an atomic or submicroscopic scale[19], it is probable that only the microscopic inelastic adjustments in localized regions are important in determining the notch sensitivity. These minute inelastic deformations (which constitute the property Moore called crackless plasticity) probably do not relieve the macro-stresses imposed by external load at an ordinary notch, but may develop localized strain-hardening effects which raise the fatigue strength and thus reduce the notch-sensitivity.

7. Cohesive Strength

Kuntze[20] doubted whether Thum's hypothesis of lowered peak stress as set forth above conformed to the experimental results, and propounded his own theory to explain notch-sensitivity by the mechanism of plastic

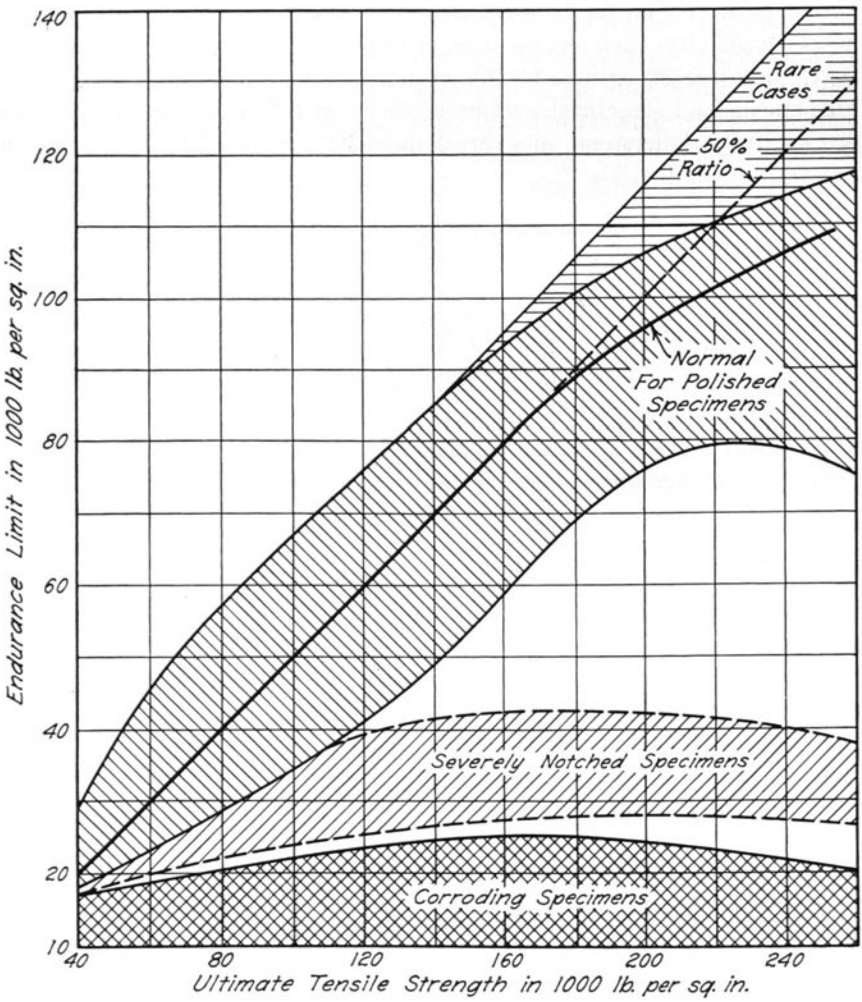

Fig. 2. General Relation Between Tensile Strength and Endurance Limit for Steel Specimens (from reference 21)

deformation. According to Kuntze, plastic deformation involved not only slip but also a loosening of small particles of materials in the test piece. In notched specimens, if the metal offered sufficient cohesive resistance, plastic slip was found to take place simultaneously throughout all the section, and the stress concentration would not influence the strength. But if the cohesive resistance was not sufficient, sliding occurred in a

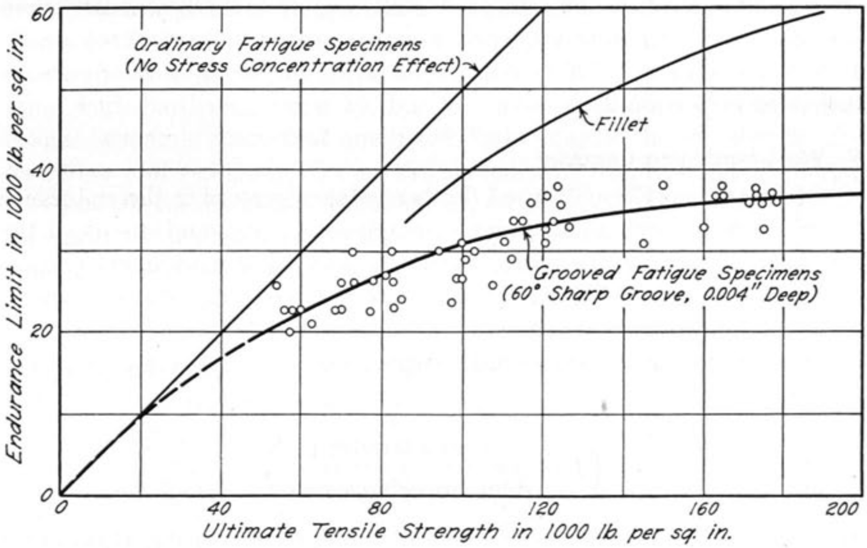

Fig. 3. Relation Between Tensile Strength and Endurance Limit for Grooved Steel Specimens (from reference 23)

portion of the cross-section only, and the material "loosened" in the remaining portion where there was no slip. Thus, according to Kuntze, notch-sensitivity of a material was explained by its relative weakness in cohesive strength.

8. Tensile Strength

Investigators(21, 22, 23, 24, 49) have shown that notch-sensitivity in fatigue of metals depends on the severity of notch and on the nature of the material, which in turn is often appraised in terms of the tensile strength. In general, the higher the tensile strength the greater is the notch-sensitivity, as shown in Figs. 2 and 3. The increased strength reduction for the stronger metals may be due mainly to their lowered capacity for the minute plastic flow which results in localized work-hardening. Consequently, the amount gained in fatigue strength for notched specimens by changing to a higher tensile strength steel varies from zero to about 100 percent depending upon the geometry of the notch; the more severe the notch, the less is the percentage gain in fatigue strength.

For steels of the same tensile strength the notch-sensitivity is altered somewhat by differences in metallurgical structure produced by different heat treatments. It was found[8] that for the same tensile strength a drastically quenched-and-tempered steel was less notch-sensitive than the same steel in a slowly quenched-and-tempered or normalized condition. Some apparent differences may also be due to residual stresses as discussed in Section 27.

9. Work-hardening Capacity

McAdam and Clyne[22] used the percentage decrease in the endurance limit due to a notch as an index of notch-sensitivity, and discussed the relationship between the notch-sensitivity and other mechanical properties of metal, namely ductility, mechanical hysteresis, tensile strength, and work-hardening capacity. It was emphasized that notch-sensitivity depended considerably on work-hardening capacity and it was proposed that the quantity

$$\left(1 - \frac{\text{tensile strength}}{\text{true breaking stress}}\right)$$

might be used as an index of tensile work-hardening capacity. However, a direct comparison between these indices of notch-sensitivity and work-hardening capacity from experimental test results showed a wide scattering and apparently did not indicate a consistent relation for different materials or for specimens of different geometry.

Gillett[25] reports that for 18:8 stainless steels the endurance limit on a polished unnotched bar in the fully soft condition was found to be less than that of a similar bar with a notch. In general, austenitic stainless steels all had surprisingly good notched fatigue strengths and were not notch-sensitive. This fact has been attributed to their remarkable work-hardening capacity as revealed in static tension tests. Some manganese austenitic steels which showed similar work-hardening behavior in static tension tests have been expected by some investigators to have the same low notch-sensitivity in fatigue, though no data have been reported.

If the work-hardening capacity of a material is really responsible for low notch-sensitivity, one may wonder why the fatigue strengths for unnotched specimens (of these austenitic steels for example) are not increased by work-hardening in the same proportion as the notched specimens. That is, do the unnotched specimens have the same work-hardening capacity as the notched specimens of the same material? This question can be answered by considering the effect of the stress gradient and the work-hardening capacity in a localized zone. The notched specimen (with a steep stress gradient) requires work-hardening of only a small localized volume to resist the peak stress applied. Inelastic deformation at the root of the notch has the effect also of reducing the peak stress and of bringing a larger volume of material into play in resisting the load; this is equivalent to a "supporting effect" from the elastic material surrounding the hardened volume. The material has the same work-hardening capacity, but the notched specimen shows higher fatigue strength than that predicted from elastic theory, due to the restraints and readjustments in the surrounding "understressed" metal.

Since the mechanism of work-hardening is not completely understood, the term "work-hardening capacity" (which refers to the maximum amount of cold work which a material can receive without fracture) is essentially a hypothetical concept which assumes that when a material reaches its work-hardening capacity fracture ensues. The nature of work-hardening is vague in simple static tension tests; in repeated loading the mechanical readjustments are presumed to be even more complicated.

III. ANALYSES OF STRESS CONDITIONS

10. Change of Notch Radius

Any elastic or plastic strain at the root of a notch tends to change the root radius. Moore and Jordan[26] assumed that fatigue loading might produce a lengthening of notch radius r over a very small length of arc at the bottom of the notch. Lengthening of the notch radius from r to r' would reduce the concentration factor K_t to an "effective" stress concentration factor K_e. From test data on two steels (SAE 1020, and quenched and tempered SAE 2345) they obtained an empirical expression for the effective radius r' which if substituted for r in Neuber's diagram[5] gave values of stress concentration factor equal to K_e as found from actual fatigue tests. This empirical expression was:

$$r' = c_1 d^{2/3} - c_2 t + r \tag{2}$$

where c_1 and c_2 were material constants determined by fitting the equation to test data; d was the net diameter of the test section; and t was the notch depth. However, tests have shown[51] that there is no appreciable change in the actual notch radius in the usual fatigue test.

11. State of Stress and Shear Energy Theory

Several investigators[27, 28, 49] considered that the notch effect might be partly or entirely due to the state of combined stress existing at the notch root; hence different theories of elastic failure which had been used to explain the effect of combined stress in static tests were applied to the notched fatigue specimens. Experimental data on the effect of combined stresses on the endurance limits of *unnotched* specimens showed that for 0.1 percent and 0.34 percent carbon steels[29, 30] and a 2 percent Ni-1 percent Cr-0.35 percent Mo steel[31] the shear energy theory agreed with the test results quite well, but for a 3½ percent Ni-Cr steel[29] only the total energy theory fitted the test results. For a cast iron[29], as in static tests, the principal stress theory indicated the best agreement with fatigue test data. For *notched* fatigue specimens of high-strength steels it was found[27, 28] that the shear energy theory agreed most closely with the test results, as is discussed later.

An application of the shear energy theory to the stress conditions existing in a notched rotating beam specimen (in which the extreme fiber at the root of the notch is subjected to completely reversed stress) may be developed using the following notations:

Let S_e = endurance limit of an unnotched specimen under uni-axial stress state (p.s.i.)

S_1, S_2, S_3 = three principal stresses at the bottom surface of the notch (longitudinal, circumferential, and radial, respectively, when loaded to the endurance limit) (p.s.i.)

S_n = endurance limit (nominal flexural stress as found from the ordinary flexural formula Mc/I at the notch root) (p.s.i.)

K_t = theoretical stress concentration factor = S_1/S_n

K_τ = theoretical strength reduction or "shear energy" factor; i.e., theoretical value of the ratio S_e/S_n

u = Poisson's ratio

The shear energy criterion for failure of material may be formulated in terms of the three principal stresses S_1, S_2, S_3 as follows:

$$(S_1 - S_2)^2 + (S_2 - S_3)^2 + (S_3 - S_1)^2 = 2S_e^2 \tag{3}$$

From Neuber's theory[5] we have

S_1 = longitudinal stress = K_tS_n

S_2 = circumferential stress = $u\,(S_1 - S_n) = uS_1\left(\frac{K_t - 1}{K_t}\right)$

S_3 = radial stress = 0 (at the surface)

Substituting these relations in Eq. 3 and simplifying we have

$$K_\tau = K_t\sqrt{1 - u\cdot\frac{K_t - 1}{K_t} + \left(u\cdot\frac{K_t - 1}{K_t}\right)^2} \tag{4}$$

If we assume $u = 0.3$ and $K_t = 2$, then from the above equation $K_\tau = 0.966K_t$. For extremely large values of K_t the value of the theoretical strength reduction factor approaches $0.954K_t$ when calculated from Eq. 4.

In 1943 both Moore and Morkovin[27] and Peterson[28] tried to employ various theories of failure of elastic action, especially the shear energy theory, to explain the observed differences between theoretical and experimentally observed strength reduction due to notches. The results of Moore and Morkovin's investigation on three SAE steels (1020 as-rolled, 1035 as-rolled, and X4130 quenched and tempered) indicated that the shear energy theory correlated with the test results better than the principal stress theory or the shearing stress theory for specimens not smaller than ½ in. in diameter. However, there were tendencies for the small specimens to behave differently from the predictions of any of these theories.

Peterson[28] analyzed Moore and Jordan's data[26] on two steels and found that the data on the quenched and tempered SAE 2345 steel could be predicted fairly accurately by the shear energy theory, but that for a low carbon steel the reduction of fatigue strength due to the notch was generally less than that indicated by the theoretically computed factors.

Comparison of the different theories of elastic failure has indicated that the principal stress theory or the shearing stress theory predict the highest values of effective stress concentration factor, i.e., these theories require that $K_r = K_t$. In order of decreasing magnitude the total energy theory requires lower values of K_r, the principal strain theory gives still lower values, and the shear energy theory predicts the lowest value for K_r. However, when actual test data for ordinary steels are compared, even the values of K_r predicted by shear energy theory are very often too high (except for high strength heat-treated steels and for large specimens which respectively are more notch-sensitive than low strength as-rolled steels or small specimens). For brittle metals of low fatigue notch-sensitivity like cast iron, none of these theories are adequate to explain the insensitivity to the stress-raising effects of a notch.

IV. ELEMENTARY STRUCTURAL UNIT

Extensive analytical work has been done to evaluate the localized stresses at notches of various shapes according to the classical theory of elasticity, and the results found were assumed to be directly applicable to mild notches or to those regions where the stress variation was not drastic[5]. As the radius of the notch approaches zero, however, the stress concentration factor theoretically approaches infinity, which is not true for actual materials.

The classical theory of elasticity assumes the material to be perfectly homogeneous and infinitely divisible, and does not recognize the structure to consist of finite particles as in the case of actual engineering materials. The fact that the actual materials are made of a finite number of particles as atoms or crystal grains of definite dimensions for each kind of material has been recognized by some mathematicians in their stress analyses; the concept was introduced that these particles might be represented by many small cubic blocks of uniform size called the "structural elementary unit." The size of the structural elementary unit was assumed as a property of the material.

If the values of the theoretical stresses in the region of peak stresses are averaged over the surface of an elementary structural unit, the value of the effective maximum elastic stress would be reduced due to the steep stress gradients existing over the unit; hence, the stress concentration factor would also depend upon the size of such a particle when the notch is sharp.

12. Stress at the End of an Elliptic Crack

Gurney[32] derived an equation mathematically for computing the average elastic stresses over the area of an elementary structural unit at the end of an elliptical hole or crack. The results are expressed in terms of the ratio ρ'/r of the length of the structural unit to the radius of curvature of the end of the elliptic crack, which has axes of lengths a and b.

In Gurney's equation if the ratio a/b remains constant, the average elastic stress (and therefore the stress concentration factor) decreases as the ratio ρ'/r increases, as is shown in Fig. 4. When r is equal to ρ', the average stress is $1.1\,a/b = 1.1\sqrt{a/\rho'}$, whereas the peak stress computed by the theory of elasticity is $2\,a/b$. When r approaches zero,

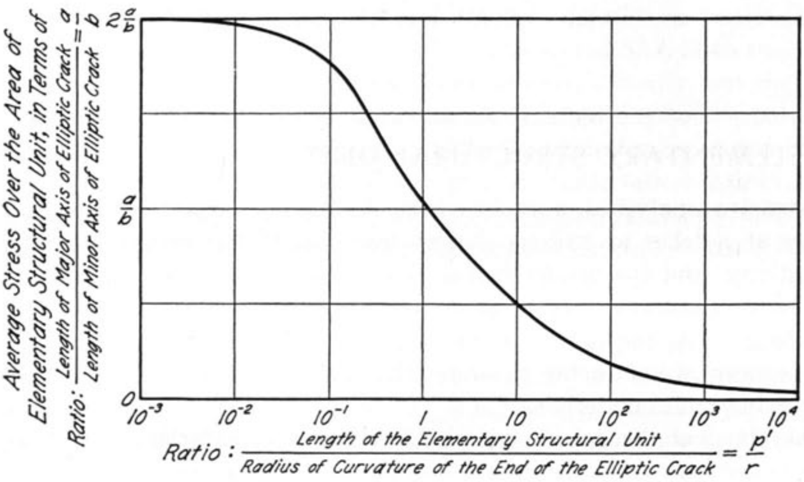

Fig. 4. Average Stress on Elementary Structural Unit in Terms of the Relative Sharpness of the Notch (from reference 32)

the average stress becomes $1.8\sqrt{a/\rho'}$ whereas the peak stress would be infinite according to the classic theory of elasticity. Therefore, the effect of an elementary structural unit of constant length $\rho' = r$ is roughly to halve the stress concentration, whereas reducing r from a value of ρ' to zero (increasing the value of K_t from $2\,a/b$ to infinity) only increases the effective stress concentration factor by about 70 percent.

13. Neuber's Formula for Sharp Notches

Neuber[5] took the effect of the size of elementary structural unit into consideration by selecting the following equation for predicting the effective stress concentration factor K_e from the theoretical value of K_t:

$$K_e = 1 + \frac{K_t - 1}{1 + \sqrt{\dfrac{\rho'}{r}}} \tag{5}$$

in which r = notch radius and ρ' = half the width of the elementary structural unit. Eq. 5 may be rewritten as:

$$\frac{K_e - 1}{K_t - 1} = \frac{1}{1 + \sqrt{\dfrac{\rho'}{r}}} \tag{5a}$$

The left member in the above equation represents notch-sensitivity index q defined by Thum and Peterson[17] (see Section 6). Hence q

appears to be a function of both the notch sharpness r and the size ρ' of the structural unit.

Neuber mentioned that for sharp notches there was a relatively large deformation at the notch root which the basic equation of elasticity did not take into consideration. Since this deformation lowered the stress concentration in the same sense as did the concept of individual structural units, to include only one of these two factors was sufficient if an empirical material constant was evaluated to fit experimental results. That is, the effect of the deformation was also included in selecting a value of ρ' in Eq. 5 to fit the actual material behavior.

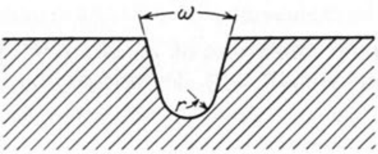

Fig. 5. Definition of Notch Geometry: Notch Angle = ω, Notch Radius = r (from reference 5)

The effect of notch angle ω (Fig. 5) was considered negligible for relatively blunt notches but not for sharp notches (i.e., those with relatively small notch radius). Neuber therefore derived the following equation from Eq. 5 to take care of the effect of notch angle:

$$K_e = 1 + \frac{K_t - 1}{1 + \dfrac{\pi}{\pi - \omega}\sqrt{\dfrac{\rho'}{r}}} \tag{6}$$

or

$$q = \frac{K_e - 1}{K_t - 1} = \frac{1}{1 + \dfrac{\pi}{\pi - \omega}\sqrt{\dfrac{\rho'}{r}}} \tag{6a}$$

By using 0.48 mm (= 0.019 in.) as the empirical value of ρ', Neuber found that Eq. 6 agreed with the results of bending tests (probably static tests) and photoelastic measurements. Since Neuber's formulas are not entirely based upon rigid mathematical analysis, but result from empirical interpolation between theoretical limiting values, the extent of its application as an exact relation is likely limited.

Furthermore, basically both Gurney's and Neuber's equations were derived only for the case of *static* loading, and only for explaining the notch effect (not including size effect). Hence it is a question how far these theories can be generalized to explain notch effect and size effect in *repeated* loading.

14. Morkovin and Moore's Application to Fatigue Test Data

Morkovin and Moore[33] found Neuber's value of $\rho' = 0.019$ in. to agree well with fatigue test results for SAE 1020 and SAE 1035 steels as-rolled. However, $\rho' = 0.0014$ in. was found by trial and error to give a better correlation with the results of fatigue tests of annealed SAE 1035 steel; a value of $\rho' = 0.00068$ in. was obtained for quenched and drawn SAE X4130 steel.

They mentioned that the test data available did not seem to justify any attempt to determine a quantitative correlation between the value of ρ' and grain size.

15. Moore's Formula for Values of ρ'

In order to determine the value of ρ', Moore[34] presented an empirical formula which, when used with Eq. 6, gave computed values which agreed fairly well with the strength reduction factor K_e from actual fatigue tests of six steels. This relation is:

$$\rho' = 0.2\left(1 - \frac{S_y}{S_u}\right)^3 \left(1 - \frac{0.05}{d}\right) \text{(inch)} \tag{7}$$

where

S_y = yield strength of the steel, p.s.i.

S_u = tensile strength of the steel, p.s.i.

d = critical diameter at the root of the notch, in.

The basis for selection of the above equation was as follows. It was thought that the increased strength indicated by the fact that K_e was less than K_t was due in large part to the resistance to fracture of a metal under repeated plastic strain. This increased resistance was arbitrarily assumed to be some function of the ratio between yield strength S_y and tensile strength S_u. The assumption that

$$\rho' = c\left(\frac{S_u - S_y}{S_u}\right)^3 = c\left(1 - \frac{S_y}{S_u}\right)^3 \tag{7a}$$

was tried first; by empirical selection of the constant c it gave favorable agreement with experimental results. Then the probable tendency of small specimens to be weaker because of the proportionally larger area occupied by a single crystal grain was considered, and a modified formula was tried:

$$\rho' = c\left(1 - \frac{S_y}{S_u}\right)^3 \left(1 - \frac{\alpha}{d}\right) \tag{7b}$$

By a process of trial and error the values of c and α were obtained as 0.2 in. and 0.05 in. respectively, to give best agreement between computed values and the values of K_e determined directly from fatigue tests.

Moore found it preferable to regard ρ' as an inverse measure of notch-sensitivity only, instead of the dimension of a structural unit as Neuber originally conceived it.

Moore's empirical formula (Eq. 7) and his method of applying it agreed reasonably well with the fatigue data for six steels he studied; a correlation with data on SAE 1045, 3140 and 2340 steels quenched and drawn to a structure of tempered martensite[8] also has been attempted during the course of this study. However, for quenched and drawn SAE 4340 steel having a ratio of S_y/S_u equal to 0.95[35], the prediction of the value of K_e by Moore's method yielded a deviation of about +20 percent; for slowly quenched-and-drawn or normalized SAE 1045, 3140 and 2340 steels having ratios of S_y/S_u from 0.62 to 0.68, and for an austempered SAE 2340 steel for which $S_y/S_u = 0.79$, the predictions deviated by about −22 to −42 percent of the observed value of K_e.

V. STRESS GRADIENT AND STRESS CONCENTRATION

The stress gradient, as represented by the slope of the stress distribution curve at the root of a notch, has been established as an important factor in notch-sensitivity[20, 36, 39]. The maximum stress gradient denoted by m p.s.i. per in. may be estimated by the following formulas. For a shaft with a transverse hole and loaded in pure bending the maximum stress gradient[36] is as follows:

$$m = 2.3 \frac{K_t S_n}{r} \tag{8}$$

And for a bending shaft with a fillet[36] it is

$$m = 2.6 \frac{K_t S_n}{r} \tag{9}$$

in which

$K_t S_n$ = theoretical maximum stress, where S_n is the nominal stress as found by elementary stress formula, p.s.i.

r = radius of hole or fillet, in.

For a circular shaft with circumferential grooves, the following equations[53] may be used:

For direct tension:
$$m = \frac{K_t S_n}{r} \cdot 6 \frac{\sqrt{\frac{t}{2r}} + 1}{3\sqrt{\frac{t}{2r}} + 2} \tag{10}$$

For pure bending:
$$m = \frac{K_t S_n}{r} \left[6 \frac{\sqrt{\frac{t}{2r}} + 1}{3\sqrt{\frac{t}{2r}} + 2} + \frac{2}{d} \right] \tag{10a}$$

where

S_n = nominal stress on net section as found by elementary stress formula, p.s.i.

r = radius of groove, in.

t = depth of notch, in. = ½ (diameter of gross section $-d$)

d = diameter of net section at root of notch, in.

It will be noted that the value of m in Eq. 10 ranges only from $2/r$ to $3/r$ times the theoretical stress K_tS_n for a wide range in values of the ratio t/r. For pure bending the stress gradient is also a function of the actual diameter of the specimen, as indicated by the last term in Eq. 10a.

16. Relative Stress Gradient

In correlating fatigue data on notched specimens, Aphanasiev[20] derived an empirical relation between the notch-sensitivity as measured by the ratio K_e/K_t and the relative stress gradient as measured by the ratio m/S_n:

$$\frac{K_t}{K_e} = \left(\frac{m}{aS_n} + 1\right)^b \tag{11}$$

where a and b are material constants; the other symbols are as defined previously. From this equation it is seen that the notch-sensitivity as measured by the ratio K_e/K_t increases as the relative stress gradient m/S_n decreases. An irrational shortcoming of the formula is that the relative stress gradient m/S_n (and consequently the material constant a) have the dimension 1/mm or 1/in.; a fundamental *constant* for the material probably should not have such a dimension without physical meaning.

17. Stress Gradient, State of Stress, and Amount of Stress Concentration

Roedel[39] made a direct comparison of notch-sensitivity index q (Eq. 1) and stress gradient m for rotating beam specimens with three different types of notches; namely, with transverse holes[17]; with fillets[17]; and with semicircular grooves[26, 27]. His results showed that for the same material the notch-sensitivity index depended upon three factors — the stress gradient; the state of stress; and the ratio r/d (i.e., the ratio of notch radius to net diameter, which was considered as an index of the stress concentration). Part of his results are shown in Fig. 6.

If the ratio r/d remained constant, Roedel found that the following results were obtained: (a) for shafts with transverse holes, the notch-sensitivity index dropped rapidly at a decreasing rate and became nearly zero for large stress gradients; (b) for a more biaxial state of stress, such as shafts with fillets, q dropped with increased stress gradient but seemed to reach a minimum value well above zero for high stress gradients; (c) for shafts with grooves, q dropped at a decreasing rate, but the minimum value of q when determined for high stress gradients was above that for shafts with fillets.

In all cases it was concluded that: (a) for very low values of stress gradient, q always approached a value of 1.0; (b) for intermediate

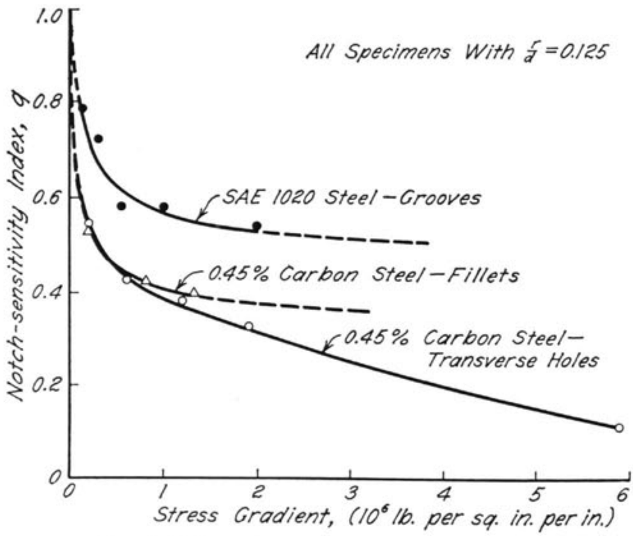

Fig. 6. Relation Between Notch-sensitivity Index and Stress Gradient (from reference 39)

values of stress gradient, q decreased as the stress gradient increased, but was also influenced by the biaxiality of the stress and by the acuity of the notch; q was higher for lower values of r/d*.

Roedel explained his results by a reasoning similar to Thum's (see Section 6) which assumed that some small amount of local plastic yielding took place to lower peak stress during repeated loading. Conditions combining high stress gradients with a small degree of triaxiality of stress led to local slip that relieves stress concentration and hence permits greater loads than those calculated from theoretical stress concentration, i.e., lower values of notch-sensitivity.

According to his analysis of plotted data, the geometric quantities found to give closest correlation with values of q for a range of values of these variables were:

r^2/d or K_t/r for shafts with grooves

r/d or K_t/r for shafts with transverse holes

r or r/d^2 for shafts with fillets.

He gave no physical meaning to these arbitrary geometric quantities.

* Since m is roughly proportional to K_tS_n/r (according to Eqs. 8 to 10 which are based upon the theory of elasticity) if m remains constant and r decreases, K_t or S_n must also decrease. Because

$$q = \frac{K_e - 1}{K_t - 1} = \frac{S_e/S_n - 1}{K_t - 1}$$

its value will increase when S_n or K_t decreases. Therefore, the statement that q is higher for lower values of r/d is logically necessary if d remains a constant.

18. Influence of Grain Size

Peterson obtained test data from specimens of carbon and alloy steels with fillets or transverse holes and tried analyzing the following two types of criteria for notch-sensitivity; both criteria are based on the stress gradient and some arbitrary measure of grain size[36].

The Number of Grains in Regions of Peak Stress. In considering the geometric effect and size effect of specimens, he contemplated that the region of peak stress in a notched specimen was of prime importance, and that a different result would be expected when one or two grains were contained in the region than would be the case if 10,000 were contained in the same region. This lead to the idea of a criterion based on the number of grains in an arbitrarily selected volume at peak stress. If (say) the region stressed to within 5 percent of peak stress was selected, the volume theoretically affected could be determined from photoelastic fringe photographs, and the number of grains per unit volume could be roughly estimated from a photomicrograph of the metal. A plotting was made on semilog paper (Fig. 7) using experimentally determined values of notch-sensitivity index as ordinates and using the number of grains within 5 percent of peak stress as abscissas on a log scale. In this manner a straight line with its scatter band was obtained which may be represented approximately by the following relation:

$$q = 0.378 + 0.141 \log g \pm 0.2 \qquad (12)$$

where g = number of grains within five percent of peak stress. The last term $\pm$ 0.2 indicates the approximate scatter band.

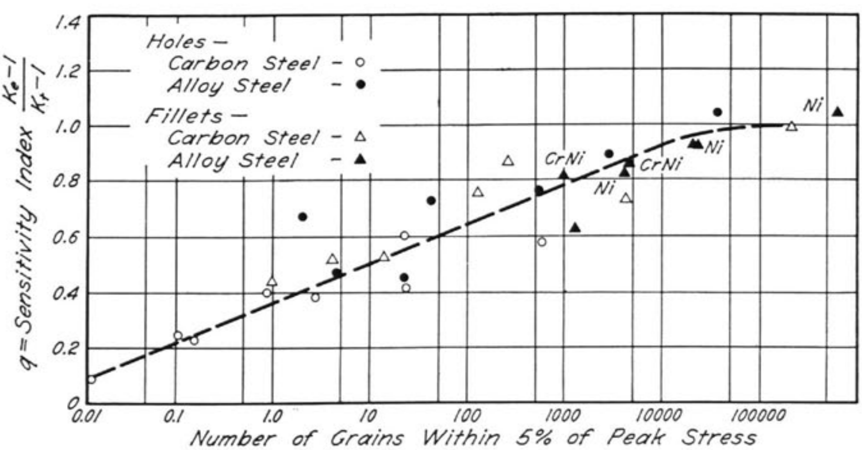

Fig. 7. Notch-sensitivity Index vs. Number of Grains in Region of Peak Stress (from reference 36)

Decrement in Stress Across One Grain. Another criterion for notch-sensitivity studied by Peterson was based on the thought that the stress gradient at the surface may be an inverse measure of the tendency of initial damage to propagate across the section. The notch-sensitivity

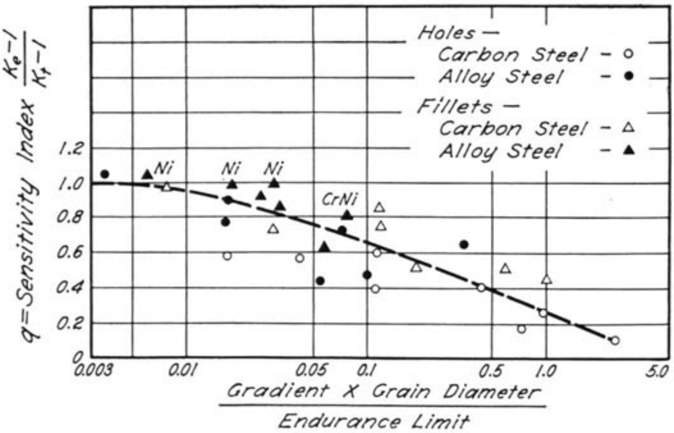

Fig. 8. Notch-sensitivity Index vs. Relative Decrement in Stress Across One Grain (from reference 36)

index was plotted against the relative decrement in stress across one grain, and a curve as shown in Fig. 8 was obtained which may be approximated by the following equation:

$$q = 0.250 - 0.358 \log S_d \pm 0.2 \tag{13}$$

where $S_d = \dfrac{\text{stress gradient at peak stress} \times \text{grain diameter}}{\text{endurance limit of the material}}$

The above two criteria do not differ markedly in numerical results; that is, both show about the same deviation with respect to the scatter of actual test data. The second criterion, however, was preferred by the author.

Later Development. In 1945 Peterson modified his manner of presenting similar test data on notch-sensitivity[37]. It was observed that over a considerable range, the stress gradients for stress raisers such as fillets, grooves and holes were approximately proportional to $1/r$. Therefore r was used as a parameter representing stress gradient and the test data were plotted on charts of q versus r (Fig. 9); the notch-sensitivity index was defined in a slightly different manner as shown by the relation:

$$q = \frac{K_e - 1}{K_\tau - 1} \tag{14}$$

in which K_e is the strength reduction factor calculated directly from test data as defined before, and K_τ is the theoretical shear energy concentration factor as defined by Eq. 4.

As shown in Fig. 9, a family of curves were obtained representing different kinds of steel with different grain sizes. The top curve represents fine-grained quenched and tempered steels; the middle pair of

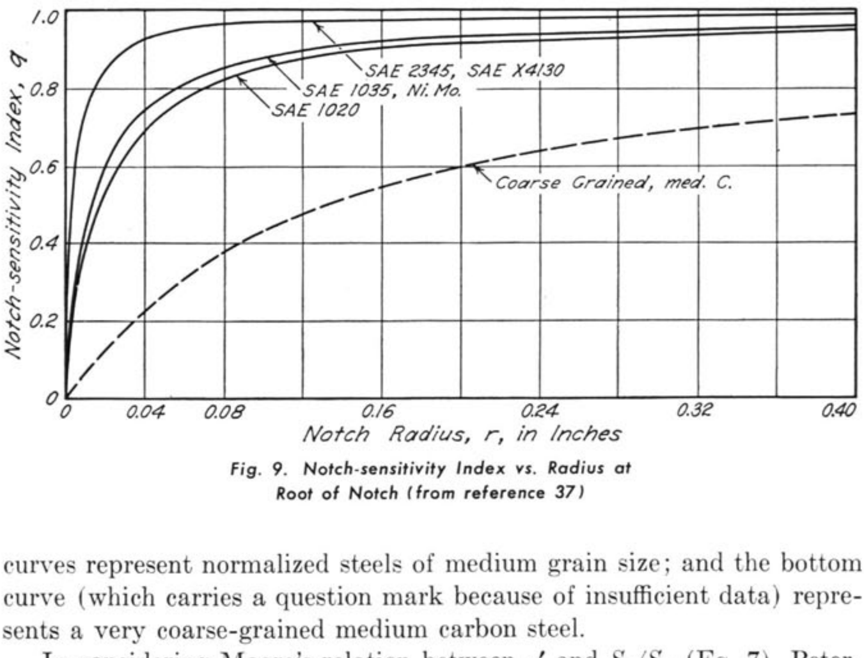

Fig. 9. Notch-sensitivity Index vs. Radius at Root of Notch (from reference 37)

curves represent normalized steels of medium grain size; and the bottom curve (which carries a question mark because of insufficient data) represents a very coarse-grained medium carbon steel.

In considering Moore's relation between ρ' and S_y/S_u (Eq. 7), Peterson suggested that a family of curves of q versus r similar to those shown in Fig. 9 could also be obtained using S_y/S_u instead of the grain size as a parameter. This chart would also tend to take care of the effects of cold work during repeated stressing and would be simpler to apply in design[40].

19. The Effect of a Stress Gradient

Roedel explained the effect of stress gradient by Thum's hypothesis of lowering of peak stress by plastic action. Dehlinger[20] in contesting Thum's hypothesis explained the nonsensitivity to notching as a slowing down in the development of fatigue cracks owing to the rapid decrease of stress from the surface towards the center of the specimen. Peterson[37] also suggested that the specimen with large decrement in stress across one grain (coarse-grained material and/or steep stress gradient) was less notch-sensitive because the fatigue crack was more difficult to get started and to propagate across the grain, and stated that this trend was confirmed by testing experience.

On the other hand, however, Aphanasiev[20] contends that the possible retardation in the development of cracks might influence only the *shape* of S-N curves, but not the endurance limit.

20. Extent of Stress Concentration

The endurance limit for notched specimens is usually greater than that predicted by the theoretical stress concentration factor; therefore the endurance limit is increased from the point of view of elastic theory. Heywood[41] explained that this increase or gain in fatigue strength was due to the fact that the region of stress concentration was very localized in extent. The increase in strength can be represented as the difference between the endurance limit S_n obtained from fatigue tests of notched specimens, and S_n' predicted by using the theoretical stress concentration factor. That is, the relative gain in fatigue strength is

$$\frac{S_n - S_n'}{S_n'} = \frac{S_e/K_e - S_e/K_t}{S_e/K_t} = \frac{K_t}{K_e} - 1 \qquad (15)$$

where S_e is the endurance limit of unnotched specimens.

Heywood expressed the extent of stress concentration (irrespective of size of specimen) in terms of $\sqrt{nr}$ where r was the root radius of the notch and n was a coefficient depending on the type of notch. Since the increase in fatigue strength was assumed to be a function of the extent of stress concentration, he proposed this formula:

$$\frac{K_t}{K_e} - 1 = \frac{M}{\sqrt{nr}} \qquad (16)$$

where M was a material constant directly proportional to the relative gain in fatigue strength. Therefore, the greater the value of M, the larger was the relative notched fatigue strength (or the smaller was the notch-sensitivity).

Heywood collected and plotted the test data on charts showing $K_t/K_e - 1$ versus $1/\sqrt{r}$, and a straight line was obtained for each kind of material and each type of notch, with a scatter band of about ± 10 percent. Figure 10 for plain carbon steels is shown here as a typical example. The data in Fig. 11 for heat-treated alloy steels showed wide

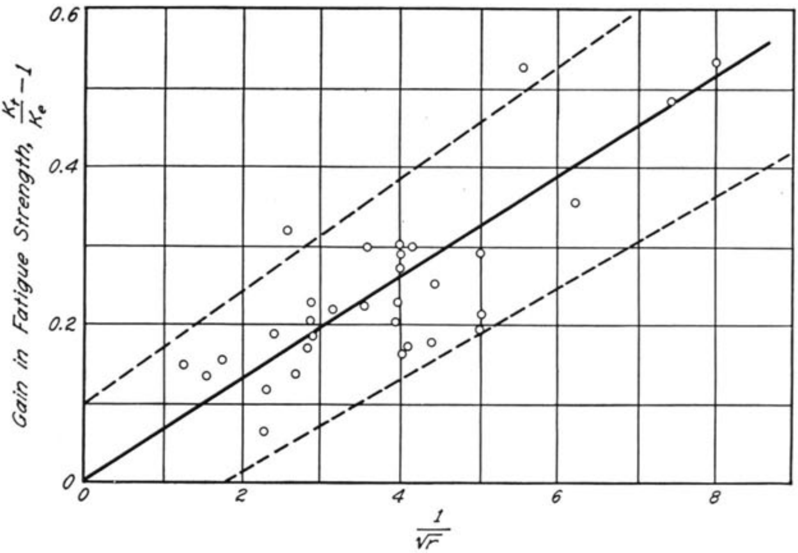

Fig. 10. Gain in Fatigue Strength vs. $1/\sqrt{r}$ for Shouldered Shaft Specimens of Plain Carbon Steels Tested in Reversed Bending: r = fillet radius in inches (from reference 41)

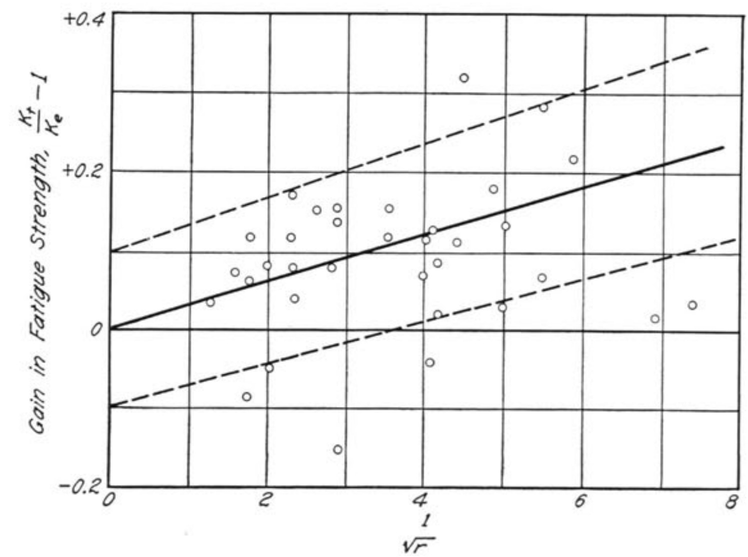

Fig. 11. Gain in Fatigue Strength vs. $1/\sqrt{r}$ for Shouldered Shaft Specimens of Heat Treated Alloy Steels: r = fillet radius in inches (from reference 41)

scatter. Apparently the heat-treated steels could not be regarded as one class of material; both chemical composition and heat treatment influenced their behavior when judged by this hypothesis. The values of M and n in Eq. 16 determined from data (such as those in Figs. 10 and 11) were as follows:

Class of Material	Values of M
Aluminum alloys	0.090
Copper alloys	0.070
Plain carbon steels	0.065
Magnesium alloys	0.044
Heat-treated alloy steels	0.030 (Average)

Types of Notch	Values of n
Shouldered shaft	1.0
Shaft with transverse hole	0.35
Vee groove in shaft	0.26

It is interesting to note the similarity between Heywood's formula (Eq. 16) and Neuber's formula (Eq. 5a), though each was derived independently and based on somewhat different concepts. To facilitate comparison, Heywood's formula and its transformations are grouped on the left and Neuber's corresponding relations are grouped on the right in the tabulation below:

Heywood's (Eq. 16)

$$\frac{K_t}{K_e} - 1 = \frac{M}{\sqrt{nr}}$$

$$\frac{K_e}{K_t} = \frac{1}{1 + \frac{M}{\sqrt{nr}}}$$

$$K_e = 1 + \frac{K_t - 1 - \frac{M}{\sqrt{nr}}}{1 + \frac{M}{\sqrt{nr}}}$$

Neuber's (Eq. 5)

$$\frac{K_t - K_e}{K_e - 1} = \frac{\sqrt{\rho'}}{\sqrt{r}}$$

$$q = \frac{K_e - 1}{K_t - 1} = \frac{1}{1 + \sqrt{\frac{\rho'}{r}}}$$

$$K_e = 1 + \frac{K_t - 1}{1 + \sqrt{\frac{\rho'}{r}}}$$

Both Heywood and Neuber interpreted the relation between K_t and K_e by a single parameter $\sqrt{r}$ for a given material and a given type of notch, but they used different relations between K_t and K_e to express notch-sensitivity. A series of six plottings were therefore made of the test data of Fig. 10 on charts of notch-sensitivity versus $1/\sqrt{r}$, using six different expressions respectively for notch-sensitivity, such as:

$$\frac{K_e - 1}{K_t - 1}; \frac{K_t - K_e}{K_e - 1}; 1 - \frac{K_e}{K_t}; K_t - K_e; \frac{K_t - K_e}{K_t K_e}; \frac{K_t - K_e}{K_e} \times \frac{K_e - 1}{K_t - 1}$$

Examples of several of these diagrams are shown in Figs. 12, 13, and 14. All these plots had similar scatter bands except points (Fig. 12) representing the relations derived from Neuber's formula. Though it fits the test data for grooved specimens of certain steels fairly well, Neuber's formula does not closely group this entire set of test data.

Heywood's formula is remarkably simple, but certain wide deviations with respect to actual test data lead to doubt of the accuracy of the formula for a wide variety of conditions. For example, the value of M is not a constant when applied to cast iron for which K_e is approximately equal to 1. Moreover, the formula would require that the gain in fatigue strength should be infinite when the notch radius approaches zero, which is contrary to actual experience.

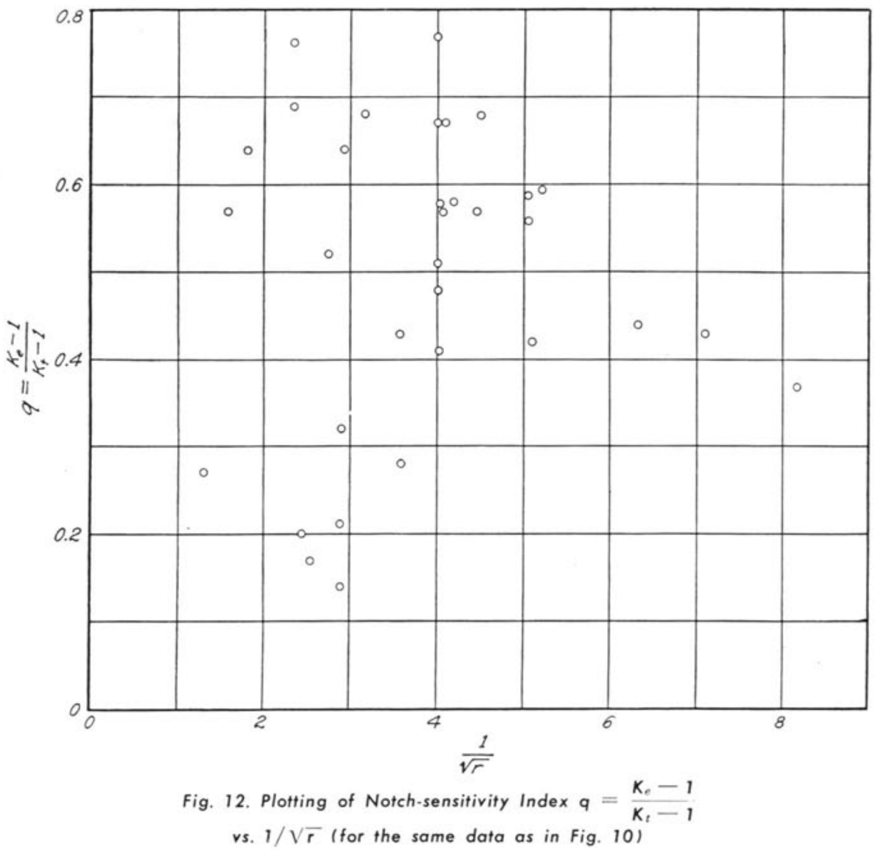

Fig. 12. Plotting of Notch-sensitivity Index $q = \frac{K_e - 1}{K_t - 1}$ *vs.* $1/\sqrt{r}$ *(for the same data as in Fig. 10)*

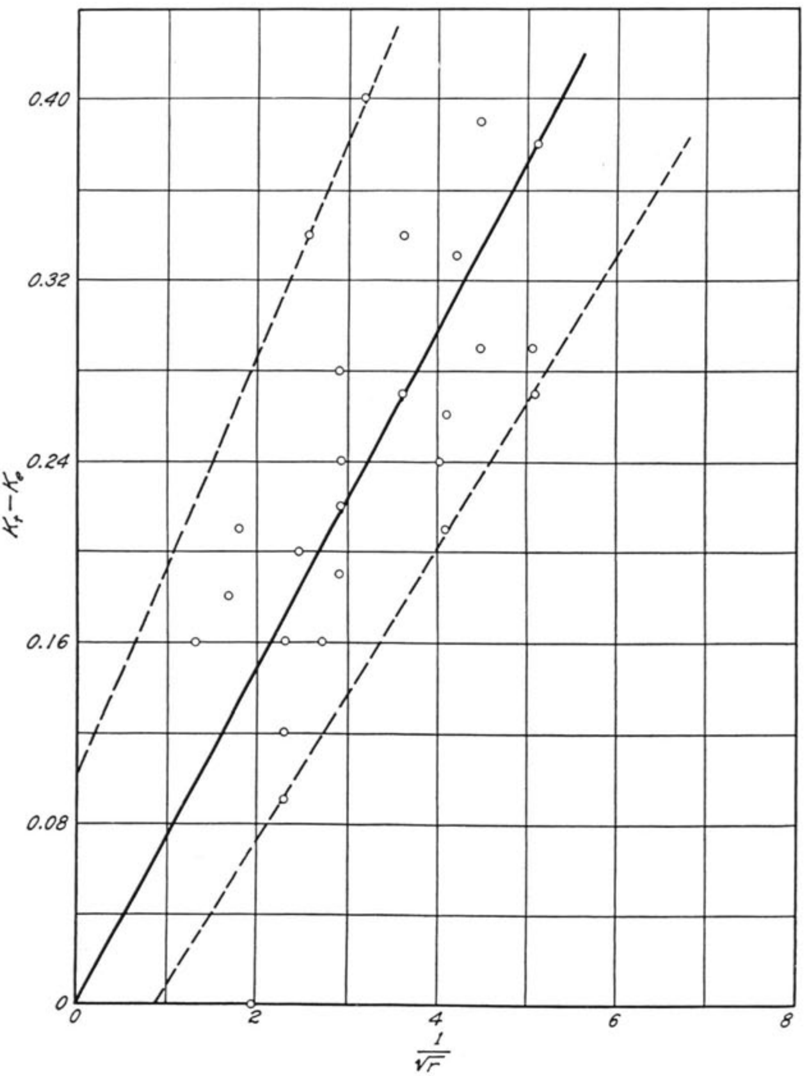

Fig. 13. Plotting of $K_t - K_e$ vs. $1/\sqrt{r}$ (for the same data as in Fig. 10)

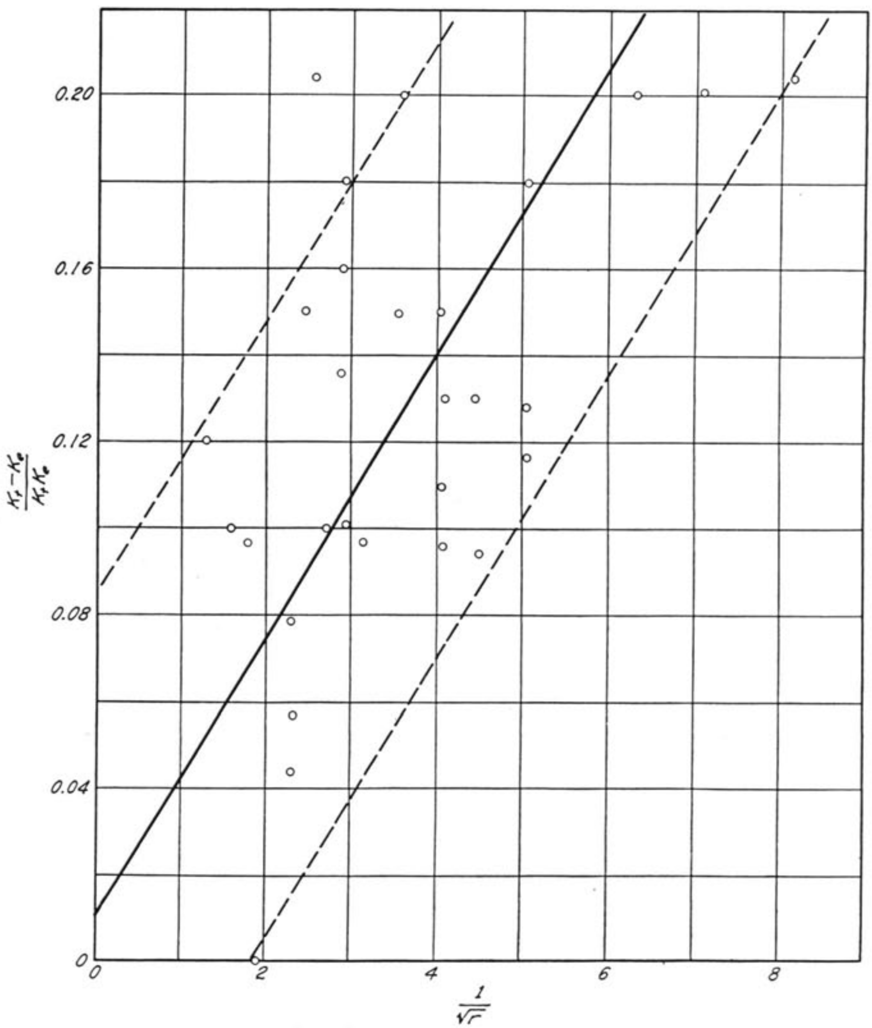

Fig. 14. Plotting of $\frac{K_t - K_e}{K_t K_e}$ vs. $1/\sqrt{r}$ (for the same data as in Fig. 10)

VI. FAILURE BELOW SURFACE

It was assumed in a hypothesis presented by Peterson[28] and Moore[34] that the fatigue strength in a bending test behaved as though the stress of significance was not that at the surface of the polished specimens but was that computed at a small distance h' below the surface. The value of h' and the computed fatigue strength S_{ec} at depth h' were assumed to be constants of the material, being independent of the size of specimen.

21. Moore and Smith's Formulas

In Fig. 15a the stress S_e at the surface denotes the endurance limit for unnotched specimens, and S_{ec} represents a critical stress at a distance h' below the surface. Both S_{ec} and h' were assumed to be basic fatigue properties for unnotched specimens independent of the diameter d of the round specimens. The following equation for *unnotched* specimens was derived in a simple mathematical procedure by Moore[34], based upon the geometry of Fig. 15a:

$$S_e = \frac{S_{ec}}{1 - \dfrac{2h'}{d}} \tag{17}$$

In a similar way Smith[35] derived the following equation for *notched* specimens from the geometry of Fig. 15b:

$$S_t = \frac{S_{tc}}{1 - \dfrac{Rh''}{d_n}} \tag{18}$$

where S_t is the theoretical maximum stress at the surface, equal to K_t times endurance limit S_n of the notched specimen (i.e., $S_t = K_tS_n$); S_{tc} is the critical stress at a distance h'' (different from h') below the surface; the value of S_{tc} is assumed to be a basic fatigue property for notched specimens independent of the diameter d_n of the specimen at the net section. R is defined as the maximum stress gradient times the diameter of the specimen divided by the peak stress ($R = md_n/S_t$). It is a dimensionless quantity and may be called the "relative stress gradient." The value of R may be computed from Eqs. 8 to 10, or may

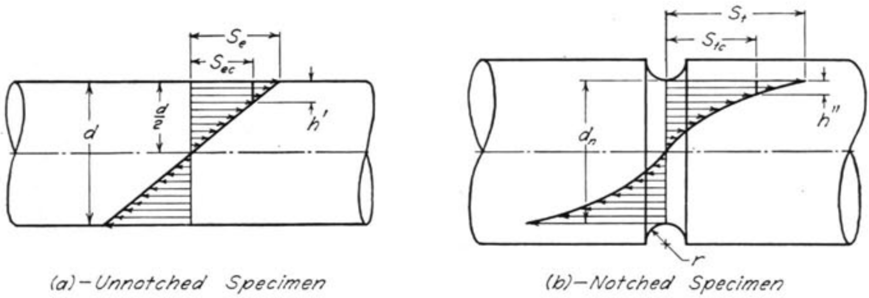

Fig. 15. Distribution of Longitudinal Stress in Rotating Beam Specimens

be derived from Neuber's theory of elasticity[5]. For unnotched rotating beam specimens $R = 2$.

Since $S_t = K_t S_n$, Eq. 18 may be rewritten:

$$S_n = \frac{S_{tc}}{K_t\left(1 - \frac{Rh''}{d_n}\right)} \tag{19}$$

From fatigue test data on six steels[34] h' was found to have an average value of 0.008 in. and S_{ec}/S_{tc} to have an average value of 0.88, so Eqs. 17 and 19 may be rewritten approximately as follows:

$$S_e = \frac{S_{ec}}{1 - \frac{0.016}{d}} \tag{20}$$

$$S_n = \frac{S_{ec}}{0.88\, K_t\left(1 - \frac{Rh''}{d_n}\right)} \tag{21}$$

By dividing Eq. 20 by Eq. 21 and letting $d_n = d$ we have

$$K_e = \frac{S_e}{S_n} = 0.88K_t \cdot \frac{1 - Rh''/d}{1 - 0.016/d} \tag{22}$$

from which the strength reduction factor K_e may be approximated.

In the above equations S_{ec} and h'' are two different constants for any given material. In order to evaluate S_{ec} and h'' for a new material, values of two endurance limits must be determined from tests of unnotched and notched bars of any size, or from tests of notched bars of *two* different diameters. After S_{ec} and h'' have been determined by substituting the test results in the above equations, the values of the endurance limits of notched or unnotched specimens of any size may be predicted from Eqs. 20, 21, or 22.

The procedure outlined above was used to calculate the strength reduction factors from Eq. 22 for grooved specimens of six steels(22) and an aluminum alloy(42) and for specimens of two steels with fillets or transverse holes(17). The deviations of the calculated values of K_e from those obtained experimentally from the ratio S_e/S_n ranged approximately from +14 to −14 percent.

The relations developed agreed with all the test data fairly well. The physical concepts or evidence and assumptions upon which this analysis was based are appraised later (Section 23).

22. Peterson's Formulas

Based upon a similar assumption of failure corresponding to the stress at constant depth h below the surface, Peterson suggested the formula(28, 40):

$$K_e = K_\tau (1 - Ch/r) \tag{23}$$

in which K_e is the strength reduction factor, and K_τ is the shear energy concentration factor as defined by Eq. 4a; C is a stress gradient constant (using $C = 3$ for grooves and fillets); and r is the notch radius. This relation appears to be a simplified empirical form of Eq. 22. Peterson(37, 40) also presented a relation similar to Eq. 23 that will give $K_e = 1$, when $r = 0$:

$$K_e = K_\tau \left[1 - \frac{Ch\,(K_\tau - 1)}{r\,(K_\tau - 1) + ChK_\tau} \right] \tag{24}$$

It can be seen that Eq. 23 and Eq. 24 are not identities. By fitting Eq. 24 to Moore's test data on SAE 1020 steels as-rolled and strain-relieved, Peterson found $h = 0.002$ in., whereas according to Eq. 22, to fit the same data, $h' = 0.008$ in. and $h'' = 0.004$ in.

23. Lack of Evidence Regarding the Basic Assumptions

Several investigators have thus discussed notch-sensitivity and its correlation with test data based on the assumption that the specimen behaves as though failure is initiated below the surface; however, no evidence has been presented to show that a fatigue fracture necessarily starts *below* the surface. It was suggested that the usual polishing may cold-work and strengthen a thin surface layer of the specimen; thus, the fracture might actually start in the weaker metal under the cold-worked layer. If this inference were correct, an SAE 1035 steel, bright-annealed after polishing[34], should not have exhibited the same size effect or notch effect; hence, this is a conclusion contrary to fact. Moreover, as pointed out by Peterson[40], if cold-working of the surface due to machining were of importance, it could often be subject to considerable variation, and the depth of failure cannot therefore be assumed constant. Furthermore, the different states of stress developed by notches of different sharpness would probably affect the depth at which failure was initiated.

It was also pointed out[34] that in all metals there are many inherent defects whose strength-reducing effect might be equivalent to that of a small notch in the surface of the specimen. Thus the fracture might start at the bottom of some inherent defect imbedded in the surface, or at some such defect slightly below the surface. However, if the inherent defects are important enough to be seriously considered, they also vary considerably in their stress-raising effects. On the basis of this concept the depth of failure can hardly be regarded as a constant, and the elementary stress formulas which lead to the above equations should not lead to useful or accurate relationships.

Some experimental evidence has been provided[43] that fatigue failure initiates in a very thin surface layer less than 0.002 in. thick and progresses inward gradually. Hence, the assumption of constant failure depth seems to be only an empirical method of correlation of a limited amount of test data for convenience in design. The assumptions made and the relations deduced are arbitrary and lack physical significance. The empirical nature of this approach was recognized by Peterson and Moore[40].

VII. STATISTICAL THEORIES OF FATIGUE

Both Freudenthal[44] and Aphanasiev[45] used statistical approaches to explain fatigue phenomena and the notch effect. While Freudenthal derived only theoretical equations for brittle metals, Aphanasiev also offered equations based on theory but modified in the light of experimental test data.

24. Freudenthal's Equation

Freudenthal considered the fatigue phenomenon as an expression of the repetitive action of an external load. This progressive destruction had the typical features of a mass phenomenon; both the cohesive bonds and the load repetitions were treated as collectives in a statistical sense. A general statistical theory was then developed by him for only truly *brittle* materials, for which no plastic deformation would occur during repeated loading so as to change the internal structure of the metal.

If a nonuniform stress distribution due to a notch or due to flexural action in a beam is approximated by a discontinuous step function with two intensities, it can be assumed that a percentage i of the total Q bonds is subjected to the high stress intensity in the vicinity of the notch, while the majority of bonds are in the field of comparatively low homogeneous stress. If p is the probability of destruction of a bond in the field of low stress intensity, and p_1 is this probability in the immediate vicinity of the stress concentration, then it was shown that the probability P_2 of rupture of all Q bonds under nonuniform stress distribution for N load repetition is:

$$P_2 = 1 - (1 - P)\, e^{-iQN(p_1 - p)} \qquad (25)$$

in which e is the base of natural logarithms, and P is the probability of rupture of all Q bonds under the homogeneous low stress, without stress concentration.

According to this equation, the probability P_2 increases rapidly with increasing value of the exponent on e, that is: (a) with increasing number of bonds $i\,Q$ in the volume affected by the stress concentration; (b) with increasing load repetitions N; and (c) with intensity of stress concentration $(p_1 - p)$. These three influences are of the same order of importance; a similar increase in the probability of rupture will be obtained either by increasing the volume affected by stress concentration or by increasing the number of load repetitions. Therefore, the effective strength reduction due to a notch should increase considerably at the lower stress levels (for which the number of load repetions is large). This agrees with Bennett's observation[46] that for SAE X4130 steel, fatigue strength reduction factors based on the endurance limits were about the same as the relative slopes of the log S-log N curves; it also agrees with Almen's suggestion[47] that the slope of an S-N curve is a relative measure of the strength reduction factor.

Freudenthal's statistical theory for brittle materials dealt only with the finite fatigue life. Therefore the endurance limit of notched or unnotched specimens, or strength reduction factor for ductile metals, cannot be predicted directly from his theory or from Eq. 25. However, his later theories[19] of the structural readjustments that occur have contributed concepts which may help to predict[50] the endurance limit of notched specimens.

25. Aphanasiev's Equation

Aphanasiev's statistical theory of fatigue[45] was based on the assumption that the metal was an aggregate of crystal grains regarded as elementary structural units having identical yield limit and cohesive strength in the direction of the external force. However, these grains were subjected to different stresses due to the inhomogeneity of the material, porosity, inclusions, influence of the grain boundaries, etc. Thus, the frequency of the occurrence of any particular stress value acting on an individual grain might be expressed as a function of the value of the imposed stress.

For fatigue loading, the critical condition or criterion for the value of the endurance limit was formulated by assuming that no grain was subjected to a stress exceeding the cohesive strength. The parameters of the probability function in the equations were evaluated from static tension test curves for the metal, which also included the effect of strain-hardening.

For the purpose of determining the fatigue strength reduction factor, the general solution presented considerable difficulties. Finally the following tentative expression for the strength reduction factor was proposed:

$$K_e = K_t \cdot \frac{1 + B \sqrt[k]{\dfrac{R_1 k + 4}{2d_1^2}}}{1 + B \sqrt[k]{\dfrac{R_2 k + 4}{2d_2^2}}} \tag{26}$$

where: $K_e = S_{e1}/S_{e2}$ = the ratio of endurance limits of specimens of two different sizes or shapes

B and k = constants of the material

d_1 and d_2 = diameters of specimens

R_1 and R_2 = relative stress gradients as defined in Section 21. ($R = 2$ for unnotched beam specimens)

This equation is adaptable to cases involving either notch effect or size effect. If applied to notched members, S_{e1}, d_1 and R_1 are for unnotched specimens, and K_t, S_{e2}, d_2 and R_2 are for notched specimens. If only the size of specimen is varied, $R_1 - R_2$ = constant; K_t = constant for both large and small specimens; and K_e represents only the ratio of endurance limits of specimens of two different sizes.

As an example, Fig. 16 was shown by Aphanasiev to indicate a good agreement of his equation with Peterson's test data[17] on bending of shafts with fillets. The curves were drawn according to the following equation as a special case of Eq. 26:

$$K_e = K_t \cdot \frac{1 + 2.2\sqrt[3]{\dfrac{1.67}{d_1^2}}}{1 + 2.2\sqrt[3]{\dfrac{R_2 + 1.33}{2d_2^2}}} \tag{26a}$$

Generally, the statistical theories of material strength apply to brittle materials only, since the complicated effect of plastic deformation changes the material structure and cannot readily be included in the analysis. Aphanasiev's theory has arbitrarily considered the effect of work-hardening, but the final relation thus depends on empirical coefficients.

There are more chances of finding a few of the weaker elements in a large specimen than in a small specimen of uniformly heterogeneous material. Hence a large specimen should be weaker than a small one according to the "weakest-link" theory (i.e., assuming the specimen to be equivalent to a chain which is no stronger than its weakest link). This conclusion is apparently in rough agreement with the test data on size effect. Another prediction from this theory is that a group of large specimens should show less deviation in their strengths (or fatigue life) than a similar group of the small specimens; however, this is apparently not in complete agreement with the limited test data available[33, 35].

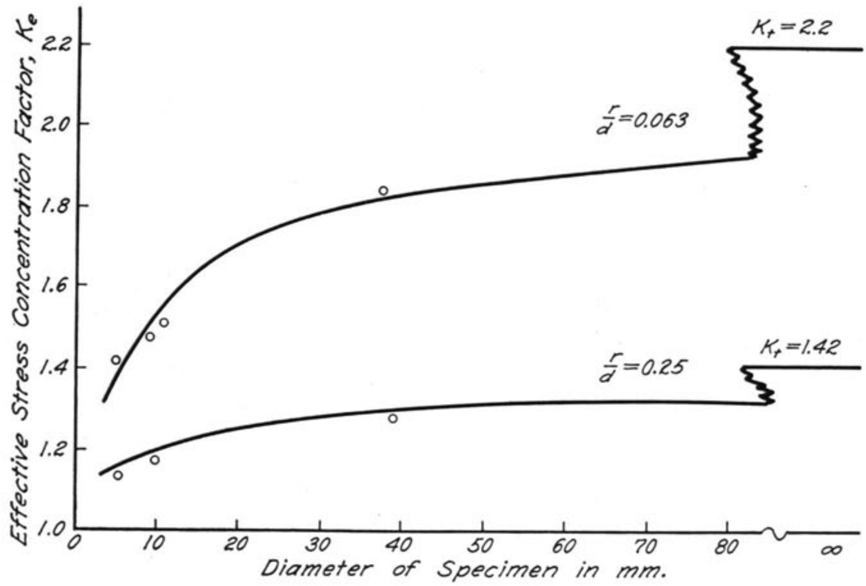

Fig. 16. Agreement of Aphanasiev's Equation with Test Data for Shouldered Shaft Specimens (from reference 45)

Apparently, the statistical or "chance effect" may be only one of the factors influencing the behavior; other contributions from inelastic deformation on a microscopic scale must be given consideration before the "notch effect" and the "size effect" can be quantitatively evaluated.

VIII. HOMOGENEITY OF MATERIALS

26. Cast Metals

Another important consideration is the effect of the homogeneity or heterogeneity of the metal on notch-sensitivity. It has been mentioned that cast iron and some cast aluminum alloys exhibit little reduction of fatigue strength due to ordinary notching of a specimen[21, 48]. This is probably related to the fact that these materials are not homogeneous even on a macroscopic scale. Cast iron, for example, may be regarded as steel full of stress raisers in the form of holes or cavities which are occupied by graphite flakes. Therefore an external notch, which is usually not as severe as the internal notches, cannot develop further pronounced reduction of strength. This case is typical for nonhomogeneous materials (or uniformly heterogeneous materials). Metals with inherent defects or internal notches like nonmetallic inclusions, blowholes, pores in sintered powders, micro-cracks, tensile residual stresses, etc., are weaker than the same metal without internal notches, but also they may not be as notch-sensitive. More experimental data are needed to confirm this general statement.

27. Heat Treatments

From the above reasoning it may be generally assumed that to make a material less homogeneous is to lower the unnotched fatigue strength relatively more than the notched fatigue strength, and hence to make the material less notch-sensitive. The more severe the unfavorable residual stresses, the less homogeneous the metal; hence it may be expected to be less notch-sensitive if residual stresses are present. This hypothetical statement offers a possible explanation for changes in notch-sensitivity due to differences in heat treatments.

In a study of the effect of metallurgical structure on fatigue strength and notch-sensitivity of steel[8], it was found that when the tensile strengths were kept constant, the steels rapidly quenched and tempered (principally tempered-martensitic structures) generally exhibited slightly higher unnotched fatigue strengths and less notch-sensitivity than the same steels slowly quenched and tempered (structures of pearlite plus ferrite). It is difficult to determine whether the drastically quenched and tempered steels or the slowly quenched and tempered steels had higher residual stresses or higher nonhomogeneity, when these pertinent factors are considered:

(a) the higher tempering temperature given the drastically quenched steels relieves more residual stress, but probably would not heal possible minute micro-cracks;

(b) the drastically quenched steels have higher flow stress which makes the relief of residual stress more difficult;

(c) the cooling process from the high tempering temperatures may also induce some micro-residual stresses, additive to or diminishing the previous stress state.

Since the drastically quenched and tempered steels were less notch-sensitive, one possible explanation might be the presence of inherent residual stresses which made the material less homogeneous.

However, the drastically quenched and tempered steels exhibited higher notched and unnotched fatigue strengths than the slowly quenched and tempered groups. As an explanation for this fact it was suggested[(54)] that the tempered martensitic steels had relatively smaller agglomerations of weak ferrite and more uniformly dispersed hardening constituents of iron carbides which offer higher resistance to slip and crystal fragmentation and hence higher fatigue strength. Larger crystals of free ferrite in the slowly quenched steels offered a smaller resistance to slip. The net result on the rapidly quenched steels was an increase in the fatigue strengths of both unnotched and notched specimens (especially the marked increase in the strength of notched specimens).

For the same quenched steels tempered at different temperatures or the same metals cold-worked to different degrees, the tensile strengths and the work-hardening capacity will be different. The work-hardening capacity probably exerts more influence upon the notch-sensitivity than the presence of residual stresses. The higher the tempering temperature for the same quenched steels, the higher is the work-hardening capacity and the lower is the notch-sensitivity, though the residual stresses are also lower.

IX. CLOSURE

After reviewing these interpretations by numerous investigators it appears evident that the fatigue notch-sensitivity of a metal member depends upon three different factors — the basic material characteristics, the degree of material homogeneity, and the geometry of the member.

As regards the basic material characteristics, attempts have been made by investigators to formulate a quantitative expression for this "constant." The concepts of tensile strength, plasticity, ductility, damping capacity, cohesive strength, work-hardening capacity, grain size, ratio of yield strength to tensile strength, and the elementary structural unit have all been used in attempts to interpret or explain notch-sensitivity; all of these properties are probably indirectly related. For example, a given material with a small grain size may be expected to have relatively high values of notch-sensitivity, tensile strength, ratio of yield strength to tensile strength, and correspondingly low values of plasticity, ductility, damping capacity, and work-hardening capacity.

The intrinsic physical nature of the mathematician's concept of an "elementary structural unit" itself is not known; hence there is a question as to whether the structural unit is related to the size of (a) the space lattice; (b) a crystallite; (c) mean free ferrite path[52]; (d) some kind of crystal grain; or (e) some other type of constituent. If not represented by a physical constituent, the structural unit has no significance in terms of the properties of actual structural units, and can only be an empirical material constant with the dimension of length (compare the material constant a in Eq. 11).

The concept of readjustments due to plastic action has provided a useful tool to explain differences in notch-sensitivity of materials qualitatively, but there is no direct quantitative evidence for this explanation from macroscopic or microscopic measurement of deformation or stress in the vicinity of an ordinary notch (Sections 6 and 10). Since fatigue phenomena originate basically on a sub-microscopic scale, it is probable that only the very minute plastic or inelastic adjustments associated with work-hardening in a localized region (and hardly detectable in ordinary direct measurements) are important factors in determining the notch-sensitivity. The insensitiveness of austenitic stainless steel to the damaging effects of a notch may be considered as an extreme case illustrating the effect of work-hardening capacity.

No method of evaluation of the work-hardening capacity of a metal under repeated stress has been widely acceptable for use. However, for some steels, such as plain carbon steels or low alloy steels, the tensile strength (Section 8) may perhaps be regarded as the simplest and the most convenient quantity for use as a rough index of work-hardening capacity. The higher the tensile strength of a metal due to severe cold work or heat treatment, the lower is the work-hardening capacity, and hence the greater is the notch-sensitivity.

For the same tensile strength there is an effect of the heat treatment on the notch-sensitivity; this effect may be due to the difference in degree of homogeneity induced by the different heat treatments. A greater degree of material homogeneity usually results in higher strength but also is accompanied by higher notch-sensitivity. The effect of a heterogeneous structure is best illustrated by cast iron which is insensitive to the effect of a notch because it has already been drastically weakened by internal defects.

As regards the geometry of the member, the notch radius is the most important single parameter in determining the notch-sensitivity as well as in controlling the theoretical stress concentration factor (see Chapter V). The larger the radius the greater is the notch-sensitivity and the smaller are the theoretical as well as the effective stress concentration factors. The second important parameter is usually the diameter at the net section for a rotating beam specimen. Thus, notch radius and the minimum diameter are the most important geometric dimensions that govern the notch-sensitivity, strength reduction, and theoretical stress distribution. Since both "size effect" and "notch effect" in fatigue are mainly the result of geometric parameters, the essential nature of both effects is probably the same.

The material properties that have been used in attempts to predict the notch-sensitivity of a material are evaluated by "large-scale" engineering measurements, whereas fatigue behavior is originally related to or governed by atomic or submicroscopic readjustments in the structure. Therefore, any predictions of notch-sensitivity directly from existing large-scale properties, such as those obtained from the tension test, can hardly be expected to be accurate in predicting fatigue behavior. For the same material, however, the relation between geometric parameters and notch-sensitivity may be found experimentally and perhaps formulated analytically. It might then be possible by analytical or empirical study to express the difference in such relations quantitatively by introducing an empirical constant for each individual material. This constant might be a better criterion for notch-sensitivity than the large-scale properties thus far investigated, especially if a rational physical significance can be assigned to the constant. This would appear to be a useful procedure for attacking the general problem of fatigue notch-sensitivity.

When the fatigue strength of an unnotched specimen is considered, it has been the general practice to disregard the effect of a stress gradient and to assume that the specimen must fail or break if the repeated stress imposed at any *point* in the specimen is above the endurance limit. But it is believed by recent investigators that the stress gradient at peak stress, or the relative extent and amount of stress concentration, is also an important factor in determining the fatigue strength. Therefore, not only the stress and strength conditions at the *point* of peak stress (or the point of maximum shear energy according to the shear energy theory) but also the conditions existing in the *region* surrounding this critical point are important factors in determining the fatigue strength. The hypothesis of the elementary structural unit assumed this region to be as large as the structural unit, and the empirical concept of failure below the surface assumed this zone to be deep enough to initiate failure below the surface. Whenever a steep stress gradient occurs in a specimen, it is essential that the criterion for fatigue failure include not only the peak stress but also the conditions existing in a region surrounding the point.

The discrepancy between theoretical and effective stress concentration factors is attributed to the basic fact that the structural action in the materials under repeated loading is different from that under static elastic loading. However, very few studies have been based upon concepts of the fundamentally distinctive structural readjustments which occur during repeated stressing. It is felt that an approach to the problem of notch-sensitivity starting from recent knowledge of the character of fatigue damage might yield an interpretation that is at once more satisfactory and more dependable.

BIBLIOGRAPHY

1. F. B. Seely, Resistance of Materials, third edition, p. 280. John Wiley and Sons, 1946.

2. M. M. Frocht, Photoelasticity. John Wiley and Sons, 1941.

3. The original publication of Wöhler's test results was in Z. Bauwesen, Vols. 10, 13, 16, and 20; a good account of his results written in English is in Engineering, Vol. 11, 1871, and a brief summary is given in W. C. Unwin's The Testing of Materials of Construction, London, 1910.

4. R. J. Roark, Formulas for Stress and Strain. McGraw-Hill Publishing Company, 1938.

5. H. Neuber, Theory of Notch Stress-Principles for Exact Stress Calculation. J. W. Edwards Brothers, Inc., Ann Arbor, Michigan, 1946.

6. A. M. Wahl and R. Beeuwkes, Jr., "Stress Concentration Produced by Holes and Notches," Transactions, American Society of Mechanical Engineers, Vol. 56, pp. 617-625, 1934.

7. George Sachs and others, "Nondestructive Measurement of Residual and Enforced Stresses by Means of X-Ray Diffraction," National Advisory Committee for Aeronautics, November 1945, Technical Note No. 987.

8. T. J. Dolan and C. S. Yen, "Some Aspects of the Effect of Metallurgical Structure on the Fatigue Strength and Notch-Sensitivity of Steel," Proceedings, American Society for Testing Materials, Vol. 48, pp. 664-695, 1948.

9. George Sachs, "Stress Raisers," Iron Age, Vol. 146, pp. 31-37, August 1940.

10. C. H. Henry and J. G. Farmer, "Investigation of Static Notched Bar Behavior of Steel with Reference to Tri-Axial Stress and Technical Cohesive Strength," Iron and Steel, Vols. 13 and 14, pp. 509-512 and 549-551, 1943.

11. George Sachs, J. D. Lubahn, and L. J. Ebert, "Notched Bar Tensile Test Characteristics of Heat Treated Low Alloy Steels," Transactions, American Society for Metals, Vol. 33, 1944.

12. George Sachs and J. D. Lubahn, "Notched Bar Tension Tests on Heat Treated Low Alloy Steels," Transactions, American Society for Metals, Vol. 31, No. 1, pp. 125-258, 1943.

13. S. A. Herres and A. F. Jones, "A Method for Evaluating Toughness of Steel," Metal Progress, September 1946, pp. 462-469.

14. H. W. Gillet, "An Engineering Approach to the Selection, Evaluation and Specification of Metallic Materials," Battelle Memorial Institute, p. 123. Reprinted from Steel, November 22, 1943–February 14, 1944.

15. H. F. Moore, "Crackless Plasticity — A New Property of Metals," Iron Age, September 10, 1931, p. 674.

16. A. Thum and W. Buchman, Dauerfestigkeit und Konstruktion. Darmstadt Technische Hochschule, Material Prufungsanstalt, Mitteilungen, Heft 1 (80 pp.), 1932.

17. R. E. Peterson and A. M. Wahl, "Two- and Three-Dimensional Cases of Stress Concentration, and Comparison with Fatigue Tests," Journal of Applied Mechanics, No. 1, p. A15, 1936.

18. O. Föppl, "The Practical Importance of the Damping Capacity of Metals, Especially Steels," Journal of Iron and Steel Institute, Vol. 134 II, p. 393, 1936.

19. A. M. Freudenthal and T. J. Dolan, The Character of Fatigue of Metals. Fourth Progress Report, ONR Contract N6-ori-71, Task Order IV, University of Illinois, 1948.

20. N. N. Aphanasiev, "The Effect of Shape and Size Factors on the Fatigue Strength," Engineer's Digest, Vol. 5, No. 3, 1948.

21. Battelle Memorial Institute, Prevention of Failure of Metals Under Repeated Stress. John Wiley and Sons, 1946.

22. D. J. McAdam, Jr., and R. W. Clyne, "Influence of Chemically and Mechanically Formed Notches on Fatigue of Metals," National Bureau of Standards Journal of Research, Vol. 13, pp. 527-572, October 1934.

23. R. E. Peterson, "Stress-Concentration Phenomena in Fatigue of Metals," Applied Mechanics Section, Transactions, American Society of Mechanical Engineers, October-December 1933, pp. 157-171.

24. R. E. Peterson, "Amount Gained in Fatigue Strength of Machine Parts by Using a Material of Higher Tensile Strength," Proceedings, Society for Experimental Stress Analysis, Vol. III, No. II, 1946.

25. H. W. Gillett, "An Engineering Approach to the Selection, Evaluation and Specification of Metallic Materials," Battelle Memorial Institute, p. 128. Reprinted from Steel, November 22, 1943–February 14, 1944.

26. H. F. Moore and R. L. Jordan, "Stress Concentration in Steel Shafts with Semi-circular Notches," Proceedings, Fifth International Congress of Applied Mechanics, pp. 188-192, 1938.

27. H. F. Moore and D. Morkovin, "Second Progress Report on the Effect of Size of Specimen on Fatigue Strength of Three Types of Steel," Proceedings, American Society for Testing Materials, Vol. 43, p. 109, 1943.

28. R. E. Peterson, "Application of Strength Concentration Factors in Design," Proceedings, Society for Experimental Stress Analysis, Vol. I, No. I, p. 118, 1943.

29. H. J. Gough and H. V. Pollard, "The Strength of Metals Under Combined Alternating Stresses," Engineering, Vol. 140, p. 566, 1935.

30. J. Nisihara and J. M. Kawamato, "The Fatigue of Steel Under Combined Bending and Torsion," Transactions, Society of Mechanical Engineers of Japan, Vol. 6, No. 24, p. S-2, 1940.

31. R. A. MacGregor and J. M. Lessells, "Combined Stress Experiments on a Nickel-Chrome-Molybdenum Steel," Proceedings, Fifth International Congress of Applied Mechanics, p. 201, 1938.

32. C. Gurney, The Effective Stress Concentration at the End of a Crack in Materials Having Atomic Constitution. RAE Report No. Mat. 10, RAE Ref: Mat/CG/83, December, 1945.

33. D. Morkovin and H. F. Moore, "Third Progress Report on the Effect of Size of Specimen on Fatigue Strength of Three Types of Steel," Proceedings, American Society for Testing Materials, Vol. 44, p. 137, 1944.

34. H. F. Moore, "A Study of Size Effect and Notch-Sensitivity in Fatigue Tests of Steel," Proceedings, American Society for Testing Materials, Vol. 45, p. 507, 1945.

35. T. J. Dolan and B. C. Hanley, The Effect of Size of Specimen on the Fatigue Strength of SAE 4340 Steel. Sixth Progress Report, ONR Contract N6-ori-71, Task Order IV, University of Illinois, 1948.

36. R. E. Peterson, "Methods of Correlating Data from Fatigue Tests of Stress Concentration Specimens," Stephen Timoshenko Sixtieth Anniversary Volume, pp. 179-183. Macmillan, 1938.

37. R. E. Peterson, "Relation Between Life Testing and Conventional Tests of Materials," American Society for Testing Materials Bulletin No. 133, pp. 9-16, 1945.

38. D. J. McAdam, Jr. and G. W. Geil, "Influence of Cyclic Stress on Corrosion Pitting of Steels in Fresh Water and Influence of Stress Corrosion on Fatigue Limit," Jour. of Research, National Bureau of Standards, Vol. 24, p. 685, 1940; or Proceedings, American Society for Testing Materials, Vol. 41, p. 696, 1941.

39. J. P. Roedel, "The Influence of Certain Factors on Notch-Sensitivity," a term paper for course T & A M 116b, University of Illinois, 1947.

40. Discussion at the end of reference 34.

41. R. B. Heywood, "The Relationship Between Fatigue and Stress Concentration," Aircraft Engineering, Vol. 19, No. 217, pp. 81-84, 1947.

42. A. F. Technical Report No. 5726, Air Materiel Command, October 1948.

43. P. Feltham, "Fatigue in Metals, A Critical Survey of Recent Research and Theories," Iron and Steel, Vol. 21, pp. 431-436, October 1948.

44. A. M. Freudenthal, "The Statistical Aspect of Fatigue of Metals," Proceedings, Royal Society, A, Vol. 187, p. 416, 1946.

45. N. N. Aphanasiev, Zhur Teknich Fiziki, No. 4, p. 349, 1941 and No. 19, p. 1553, 1940; or see reference 20.

46. J. A. Bennett, "Effect of Fatigue-Stressing Short of Failure on Some Typical Aircraft Steels," National Advisory Committee for Aeronautics, Technical Note No. 992, October 1945.

47. J. O. Almen, "Shot Blasting to Increase Fatigue Resistance," SAE J. Trans. Vol. 51, No. 7, pp. 248-268, July 1943.

48. C. G. Goetzel and R. P. Seelig, "Fatigue of Porous Metals," Proceedings, American Society for Testing Materials, Vol. 40, p. 746, 1940.

49. Melbourne University Press, The Failure of Metals by Fatigue, pp. 309-335. December 1946.

50. C. S. Yen, A Study of Notch-Sensitivity in Fatigue of Metals. Ph.D. thesis, Department of T & A M, University of Illinois, 1949.

51. C. W. MacGregor and N. Grossman, Some New Aspects of Fatigue of Metals Brought Out by Brittle Transition Temperature Tests. Welding Research Council, March 1948.

52. M. Gensamer, E. B. Pearsall, W. S. Pellini, and J. R. Low, Jr., "The Tensile Properties of Pearlite, Bainite, and Spheroidite," Transactions, American Society for Metals, December 1942.

53. F. B. Seely and J. O. Smith, Advanced Mechanics of Materials. John Wiley and Sons, in press.

54. G. M. Sinclair and T. J. Dolan, "Some Effects of Austenitic Grain Size and Metallurgical Structure on the Mechanical Properties of Steel," Proceedings, American Society for Testing Materials, pp. 587-618, 1950.

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