May 25, 1965 N. MAROPIS **3,184,842**

METHOD AND APPARATUS FOR DELIVERING VIBRATORY ENERGY

Filed Aug. 3, 1961 3 Sheets-Sheet 1

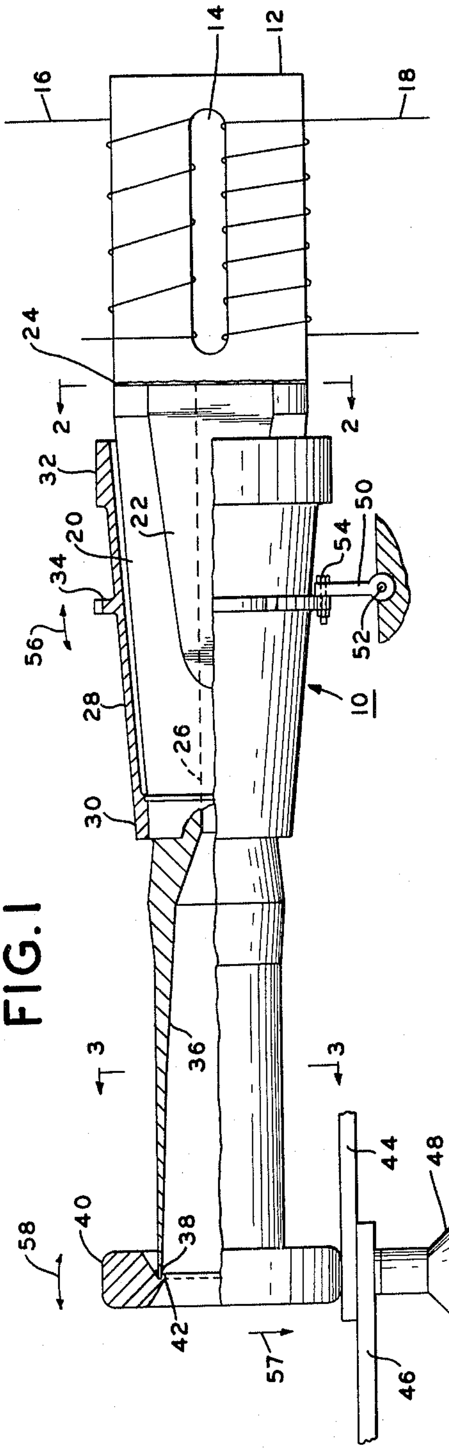

FIG.1

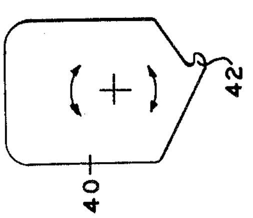

FIG.4

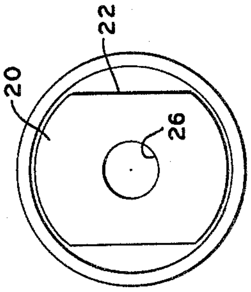

FIG.2

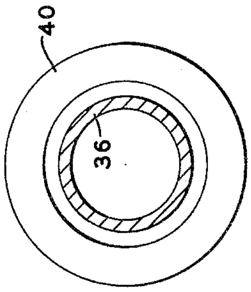

FIG.3

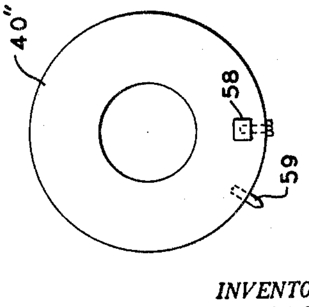

FIG.10

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METHOD AND APPARATUS FOR DELIVERING VIBRATORY ENERGY

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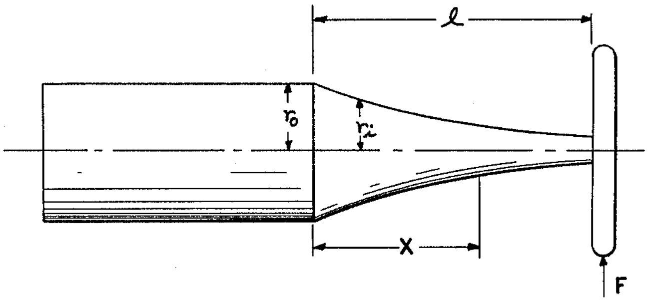


FIG. 5

FIG. 6

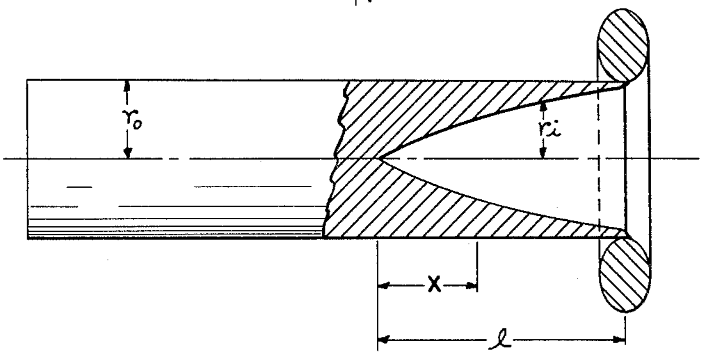


FIG. 7

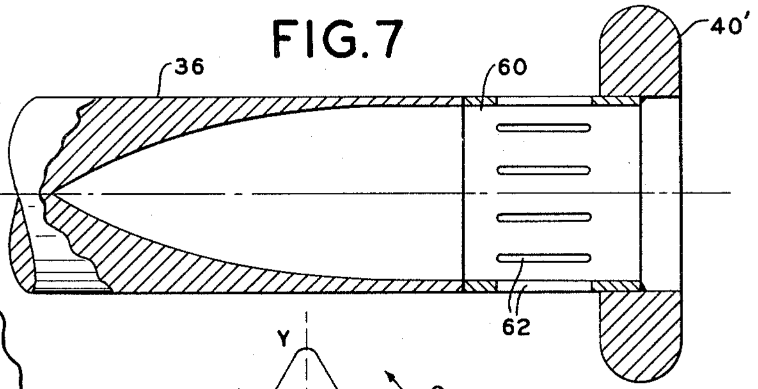
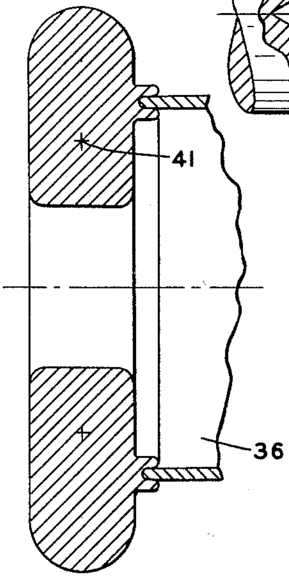


FIG. 11



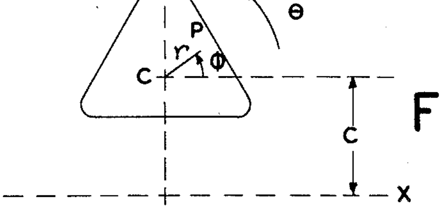


FIG. 12

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METHOD AND APPARATUS FOR DELIVERING VIBRATORY ENERGY

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FIG. 8

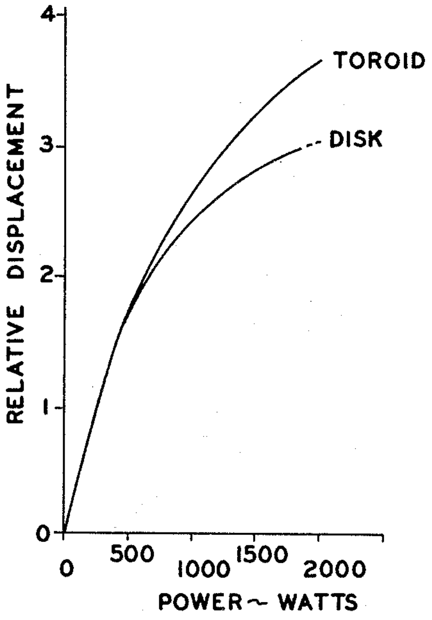


FIG. 9

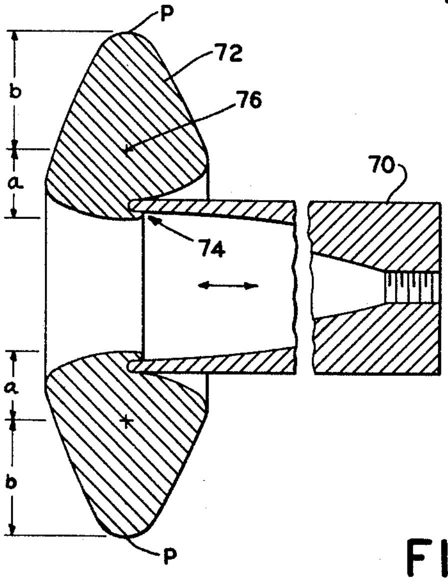
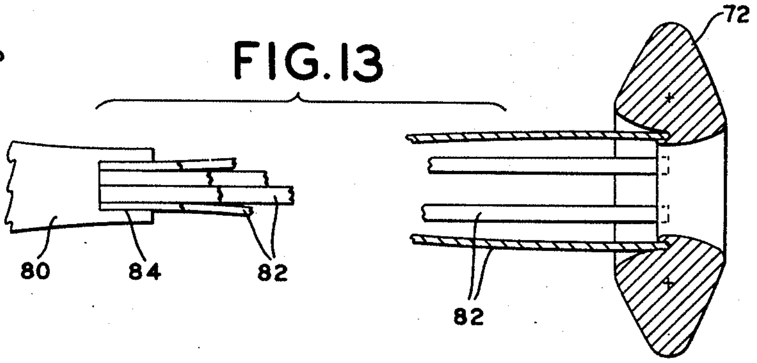


FIG. 13



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3,184,842
Patented May 25, 1965

3,184,842
METHOD AND APPARATUS FOR DELIVERING VIBRATORY ENERGY

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Filed Aug. 3, 1961, Ser. No. 129,105
15 Claims. (Cl. 29—470)

This invention relates to a method and apparatus for delivering vibratory energy. More particularly, this invention relates to a method and apparatus wherein longitudinal vibrations are converted into torsional vibrations for use in such applications of vibratory energy as welding, drilling, homogenization, cleaning, soldering, etc.

This invention has as an object the provision of a novel method and apparatus for delivering vibratory energy.

This invention has as another object the provision of novel vibratory apparatus having a resonant toroid terminal element.

This invention has as yet another object the provision of novel vibratory apparatus having a resonant toroid element connected to a resonant coupling element.

This invention has as still another object the provision of novel vibratory apparatus wherein longitudinal vibrations in a resonant coupling element are converted into torsional vibrations in a toroid type resonant terminal element.

This invention has as a further object the provision of a novel apparatus and method for effecting continuous seam or intermittent-seam weldments by the utilization of vibratory energy.

Other objects will appear hereinafter.

For the purpose of illustrating the invention there is shown in the drawings forms which are presently preferred; it being understood, however, that this invention is not limited to the precise arrangements and instrumentalities shown.

FIGURE 1 is a partial longitudinal sectional view of a vibratory device of the present invention.

FIGURE 2 is a sectional view taken along the lines 2—2 in FIGURE 1.

FIGURE 3 is a sectional view taken along the lines 3—3 in FIGURE 1.

FIGURE 4 is a transverse sectional view of the toroid.

FIGURE 5 is an elevational view of a conventional exponential coupler having a resonant disc secured to one end thereof.

FIGURE 6 is an elevational view partly in section of an inverted exponential coupler having a resonant toroid secured to one end thereof.

FIGURE 7 is an elevational view partly in section of an alternative embodiment of an inverted exponential coupler having a resonant toroid secured to one end thereof.

FIGURE 8 is a graph of relative displacement as a function of electrical power for a 2 kw., 15 kc. disc and toroid seam welding system at a typical welding condition.

FIGURE 9 is a longitudinal sectional view of an alternative coupling arrangement between an inverted exponential coupler and a toroid.

FIGURE 10 is an end elevational view of an alternative embodiment wherein the toroid has cutting tools affixed thereto.

FIGURE 11 is a longitudinal sectional view of an alternative coupling arrangement between an inverted exponential coupler and a toroid.

FIGURE 12 is a schematic diagram illustrating an asymmetrical toroidal resonant element.

FIGURE 13 is a longitudinal sectional view of another embodiment of the present invention.

In U.S. patent application Serial No. 747,254, filed July 8, 1958, entitled "Vibratory Device," in the names of William C. Elmore and Carmine F. De Prisco, now Patent 3,017,792, there is disclosed a resonant ultrasonic transducer-coupling system at whose end is a separately resonant circular plate, said plate vibrating in a normal mode of vibration symmetrically about its axis, as, for example, with one nodal circle located approximately midway between the center of the circular plate and its outer edge or periphery. This apparatus is highly effective for the purposes indicated in said application, but certain problems are encountered. Thus, there are joining and impedance matching problems associated with making metallurgical joints between the center of the resonant terminal circular plate and the coupling member to which it is to be attached. Making the coupling member and disc from a single piece does not solve these and other problems. Particularly in high-speed high-power continuous seam welding, which is resorted to for increased production output and/or improving the exterior surface condition of the weldment, the thickness and material properties of the disc set an upper limit on power delivery.

Thus, for any disc thickness there is a maximum deflection beyond which the surface stresses will exceed the elastic limit of the material, and the material of the circular plate may be stressed into its high hysteresis range as evidenced by undesirable heating of the circular disc tip and failures in those disc tips operating at the higher power levels. Possible methods of reducing the surface stresses at a given frequency including reducing disc thickness, using a material that will transmit sound at a very high velocity, or reducing the disc radius. There are practical objections to all of these approaches.

It has been found that these problems may be circumvented or alleviated by the vibratory device of the present invention, as both the joining problem and the stress problem are overcome or minimized. Thus, high acoustical powers for welding may be used as desired, for example, in connection with welding harder or thicker materials and thereby extending the usefulness of vibratory welding. Also, welding at high speeds is improved, since increased vibratory welding speed requires the use of increased powers, and the device of the present invention can handle such powers more readily than can the device of the above-identified patent application. Moreover, the coupling member to which the toroid[1] tip of the present invention is attached provides a much greater bending stiffness than does the coupling member indicated in the said application.

The present invention enables high clamping forces, which are desirable in welding at higher speeds and/or welding harder and thicker materials, to be applied. Also, whereas the resonant circular nodal-circle disc tip of the said application sometimes requires cooling in order to produce satisfactory weld, the cooling problem is alleviated or eliminated by means of the present invention. In addition, the joints between the coupling member and toroid member of the present invention are usually more easily and effectively accomplished than is the joint between the coupling member and resonant disc tip of the aforesaid patent application.

Referring to the drawings in detail, wherein like numerals indicate like elements, there is shown in FIGURE 1 a welding apparatus designated generally as **10**.

The welding apparatus **10** includes a transducer **12**. The transducer **12** comprises a laminated core of nickel

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[1] The term "toroid" ordinarily implies a circular cross-section; for the purposes of the present invention, however, it is used to include any closed loop of circular, rectangular, triangular, or other polygonal section capable of handling acoustical energy.

or other magnetostrictive metallic material. The transducer 12 is provided with a slot 14. A polarizing coil 16 and an excitation coil 18 are wound through the slot 14. Upon variations of the magnetic field strength of the excitation coil 18, there will be produced concomitant variations in the dimensions of the transducer 12, provided the polarizing coil 16 is charged at a suitable level with D.C. current.

It is to be understood that the transducer 12 is properly dimensioned so as to insure axial resonance with the frequency of the alternating current applied thereto, so as to cause it to change in length according to its coefficient of magnetostriction. In place of the transducer 12, other magnetostrictive materials may be used such as the alloy 2-V Permendur, Alfenol, etc. Also, the transducer 12 may comprise a piezoelectric material such as quartz crystals, or an electrostrictive material such as barium titanate, lead zirconate, etc.

A coupler bar 20 having flats 22 is axially joined to the transducer 12 by any conventional means such as brazing 24. The coupler 20 is provided with a longitudinal bore 26. The external surface of the coupler 20 in this instance tapers inwardly from the brazing 24 to a point substantially intermediate the ends of the coupler 20. A force-insensitive mount is provided for the coupler bar 20.

The force-insensitive mount comprises a tapered sleeve 28 fixedly secured at end 30 to the coupler bar 20. The end 32 of the sleeve 28 is free from securement with the coupler bar 20. A radially outwardly directed flange 34 is provided on the sleeve 28 intermediate the ends 30 and 32. The force-insensitive mount, which is dimensioned to operate at the nominal frequency of the device, permits the apparatus 10 to be mounted on an external member without transmitting any appreciable amount of vibratory energy to the external member. The device will operate without it, but its inclusion is preferred for best operation, especially in applications requiring the use of force.

The flange 34 on the sleeve 28 is positioned at the node one-quarter wavelength from the fixed end 30, which is also an acoustical (though not physical) one-quarter wavelength from the free end 32 (see U.S. Patent 2,891,178). The sleeve 28 comprises a cylindrical tapered metal shell, such as a cylindrical steel shell or a shell of other suitable material. The sleeve 28 has a length of one-half wavelength according to the metal used at the applied frequency, or a length equal to a unit number of one-half wavelength. The end 30 on the sleeve 28 is metallurgically bonded to a cylindrical portion of the coupler bar 20 at an anti-node or loop on the coupler bar 20, which is a zone of minimum stress and maximum particle displacement along the one-half wavelength section from 24 to 30.

The end 30 on the sleeve 28 is bonded to a cylindrical portion on the coupler bar 20 at a point substantially equidistant from the ends of the coupler bar 20. From the cylindrical portion on a coupler bar 20 to the free end of the coupler bar 20, the bore 26 is sculptured as shown at 36. Thus, the free end 38 of the coupler bar 20 is annular in transverse cross section.

A toroid 40 is metallurgically bonded to the end 38 on the coupler bar 20. The inner peripheral surface of the toroid 40 is provided with a lip 42 defining a curved socket into which the end 38 is received. Other joining schemes, such as threading, threading and brazing, or welding can be used. It will be noted that in the case illustrated the end 38 is bonded to the toroid 40 on its inner peripheral surface.

The coupler bar 20, here shown as one wavelength, is dimensioned so as to have a length equal to one-half wavelength, or unit multiples thereof, and is designed so as to be resonant at the applied operating frequency so as to deliver the maximum amount of power. The toroid 40 is also resonant at the applied operating frequency, though it vibrates in a different mode of vibration than that of the coupler bar, and does not alter the mass distribution of the coupler bar. The mode of vibration of the toroid 40 is shown by the arrows in FIGURE 4. The metal members 44 and 46 which undergo welding are disposed between the resonant toroid 40 and an anvil 48. The metal members 44 and 46 are maintained in intimate contact with each other by the force exerted on them by their engagement with the outer peripheral surface on the resonant toroid 40. Such force may be supplied by any suitable means, such as a spring means, compressed air cylinder, hydraulic cylinder means and the like engaged with the force-insensitive mount for the coupler bar 20. As seen more clearly in FIGURE 1, the flange 34 on the sleeve 28 is secured to an arm 50 by a bolt 54. The arm 50 pivots about the pin 52 as shown by the arrow 56 so that the force applied to members 44 and 46 is applied in the direction of arrow 57. Thus, the present invention is adapted to form a spot weld as well as a seam weld, and may be particularly adapted to portable type welders.

The end portion of the coupler bar 20 is internally sculptured so that the metal cross-section areas between about the sleeve end 30 and the coupler bar end 38 vary on a taper or exponentially as is well known for mechanical vibratory impedance or velocity transformers. Use of this type of coupler is not only convenient for joining to the toroid, but analysis has shown that it will accommodate greater clamping force than the ordinary exponential coupler, as the superposed oscillating fiber stress is less at the cross-section of maximum static fiber stress. The axial vibration generated by the transducer 12 in the coupler 20 is delivered to the resonant toroid 40 on its internal periphery. Depending upon the toroid and longitudinal cross-sections and impedance characteristics, the attachment of the coupler 36 can be made intermediate the I.D. and O.D. as shown in FIGURE 11. In FIGURE 11, the coupler 36 is secured to a face of the toroid between the outer periphery and the center of mass 41 of the toroid cross-section.

Vibration in torsion as here used defines a vibration in the toroidal cross-section wherein every element of this cross-section undergoes simultaneous rotation about the center of motion and through the same angle. That is, the delivery of longitudinal vibration to the inner periphery of the toroid 40 causes the outer periphery of the toroid 40 to oscillate in the direction of arrow 58 and 180 degrees out of phase with the motion on its inner periphery. The resonant frequency of the torsional vibration of the toroid 40 will be equal to the frequency of the alternating current flowing in the excitation coil 18, and to the resonant frequency of the transducer 12, coupler 20, and the mount 28, if the toroid is properly dimensioned. A theoretical analysis for this mode of vibration in either circular or non-circular cross-sections has been worked out and is disclosed hereinafter. As with other types of resonant components of ultrasonic arrays, the operating frequency of the toroid tip of the present invention is governed principally by the dimensions of that tip and the material of which it is made, in relation to the vibration principle involved. In this case, the control of frequency can be achieved by varying either the outer diameter or the thickness of the toroid; for example, resonant frequency increases with incremental increases in outer diameter and decreases with decreasing thickness.

Since the toroid 40 shown is circular in transverse cross-section, the toroid 40 is suitably adapted for seam welding when one of the contacting metal members to be welded is engaged with part of said toroid's periphery. However, it will be apparent as aforesaid that the toroid may have a variety of cross-sections and still be suitable for seam welding, such as continuous-seam or intermittent-seam welding.

Vibratory welding has been described in U.S. Patents 2,946,119 and 2,946,120. The disclosures in said patents are incorporated into the present patent application and made a part hereof. Welding in accordance with the present invention employing vibratory energy may be effected under the conditions heretofore generally developed and set forth in the said patents.

Welding is effected under a clamping force sufficient to hold the metal members being welded in intimate contact at the intended weld interface and to couple vibratory energy into the weldment. The clamping force may be varied over a wide range. In a preferred embodiment, the maximum clamping forces need not produce an external deformation of more than ten percent in weldments effected at room or ambient temperatures. By deformation is meant the change in dimensions of the weldment adjacent the weld zone divided by the aggregate thickness of the weldment members prior to welding; result multiplied by one hundred to obtain percentage. In many cases the extent of deformation is appreciably below ten percent and in many instances may be virtually absent altogether. The range of operative clamping pressures which may be employed may be readily ascertained by the user of the apparatus or method. In all cases the clamping force must be sufficient to effect coupling between the metal members being welded and the source of vibratory energy, so that such vibratory energy may be transmitted to the metal members.

The welding process may be applied to a variety of metals and alloys, examples of which include aluminum to aluminum; aluminum alloy to aluminum alloy; nickel-plated low-carbon steel to gold-plated iron-nickel-cobalt alloy; copper to copper; etc. The welding of most metals can be effected in the ambient temperature. However, the process comprehends welding under vacuum conditions or in select conditions such as atmospheres comprising an inert gas.

The welding process may be effected with metals, such as aluminum, without the extensive pre-cleaning required to effect satisfactory welding by other methods. A degree of pre-cleaning and surface treatment may prove advantageous in the ultrasonic welding of many metals. It is desirable prior to effecting welding in accordance with the present invention to remove surface contaminants, such as hydrocarbons and other lubricants and the like.

The operative range of vibratory welding frequencies which may be used includes frequencies within the range 59 to 300,000 cycles per second, and the optimum operating frequency range lying between about 4,000 and 75,000 cycles per second. This optimum range of operating frequencies may be readily achieved by transducer elements of known design, which are capable of generating elastic vibrations of high intensity.

Welding may be and in most instances is initiated at room or ambient temperatures without the application of heat. However, the weldments may be warm to the touch after the weld due to the application of the elastic vibratory energy. If desired, welding may also be initiated at elevated temperatures below the fusion temperature (melting point or solidus temperature) of any of the pieces being bonded. The temperatures to which the foregoing statements refer are those which can be measured by burying diminutive thermocouples in the weld zone prior to welding as well as the temperatures which can be estimated or approximated from a metallographic examination of a cross-section of a vibratory weld in the ordinary magnification range up to about 500 diameters. Heating the metal members to be welded prior to, and/or during welding to a temperature below their fusion temperature may, in some cases, facilitate welding and lower the power requirements and/or time requisite for welding or increase seam welding speed.

It will be appreciated by those skilled in the art that the vibratory device of the present invention may be used, not only for vibratory welding off a portion of the toroid's periphery, but also for vibratory drilling or grinding either off the periphery or off the front outer face of the toroid, which in that instance will generally operate into a liquid such as an abrasive slurry so as to produce cavitation of the liquid, if desired, to enhance said drilling or grinding performance. Operation off the front face may be also conducted without the intermediate slurry so that the face is in direct contact with the work; however, in this instance, the face of the toroid should not be employed with any great force to the work since it will not then operate in the vibrational manner desired.

Also, the toroid may be utilized for vibratory cleaning, homogenization, and other applications involving other types of liquid, with or without reference to the production of cavitation.

It is also possible to attach cutting tools to a toroid for machining purposes. See FIGURE 10 wherein the toroid **40** has a cutting tool **58** held on its front face by a screw fastener and a cutting tool **59** on its periphery. If desired, one of the cutting tools **58** and **59** may be eliminated.

The following analysis gives a comparison of the clamping forces possible with the conventional exponential coupler shown in FIGURE 5 and the inverted exponential coupler shown in FIGURES 1 and 6. In each case, the position of maximum fiber stress is computed and the ratio of the clamping forces for equal fiber stress is obtained.

From ordinary mechanics, the maximum fiber stress at any section of the system of FIGURE 5 is:

$$\sigma_{\max}=\frac{Mc}{I}=\frac{F\bar{x}c}{I} \qquad \text{(V-3)}$$

where

σ max.=maximum surface fiber stress
$M=Fx$=bending moment at any section defined by $\bar{x}$
I=moment of inertia
c=distance from center of gravity to outermost fiber

$I_L=\frac{r^4}{4}$ (where r is radius)
$\underline{C}_L=r$
$\bar{x}=l-x$

$$F_L=\frac{\sigma_{\max}I}{c\bar{x}}=\frac{\sigma_{\max}\frac{r^4}{4}}{r(l-x)} \qquad \text{(V-4)}$$

where the subscript L is associated with the lateral drive.

With regard to the variables for the toroid drive system shown in FIGURE 6, where:

$I_t=\frac{\pi}{4}(r_0^4-r_i^4)$

$C_t=r_0$

$\bar{x}=l-x$

$$F_t=\frac{\sigma_{\max}\frac{\pi}{4}(r_0^4-r_i^4)}{r_0(l-x)} \qquad \text{(V-5)}$$

where the subscript t denotes association with the toroid system.

For the standard horn of FIGURE 5 Eq. V–4 gives

$$\sigma_{\max}=\frac{4}{\pi}\frac{(l-x}{r^3}F_L \qquad \text{(V-6)}$$

for the maximum fiber stress.

In the case of the exponential horn, the radius decreases according to the equation:

$$r=r_0e-\frac{\gamma x}{2}$$

where $\gamma/2$ is the radius taper constant and corresponds with T of Equation V–17 on page 18, so that

$$\sigma_{\max}=\frac{F_L\frac{4}{\pi}(l-x)e\frac{3\gamma x}{2}}{r_0^3}$$

The position of maximum stress occurs at the position where $d_{\max}/dx=0$. This gives the position

$$X_L=l-\frac{2}{3\gamma}=l\left(1-\frac{2}{3\gamma l}\right) \qquad \text{(V-7)}$$

The maximum stress at this point is

$$(\sigma_{\max})_L=\frac{8e\left(\frac{3\gamma l}{2}-1\right)lF_L}{3\pi(\gamma l)r_0^3} \qquad \text{(V-8)}$$

For the inverted horn, Eq. V–5 shows that

$$\sigma_{\max}=\frac{4}{\pi}\frac{r_0(l-x)}{r_0^4-r_i^4}F_t \qquad \text{(V-9)}$$

If the area at any position x is the same as in the standard horn,

$$r_0^2-r_i^2=r_0^2e^{-\gamma x}$$

Therefore

$$r_i^4=r_0^4(1-e^{-\gamma x})^2$$

and

$$\sigma_{\max}=\frac{4}{\pi}\frac{(l-z)}{r_0^3(2e^{-\gamma x}-e^{-2\gamma x})}F_t \qquad \text{(V-10)}$$

The position of maximum stress is given by the solution of the transcendental equation

$$l-x_t=\frac{1-\frac{1}{2}e^{-\gamma x}}{(l\gamma)(1-e^{-\gamma x})}l \qquad \text{(V-11)}$$

This solution must be carried out for a numerical value of γl.

In the case of the horns used in the present application, γl—2.9, and it is found that

$$x_t=0.09l \qquad \text{(V-12)}$$

At this position it is found that

$$(\sigma_{\max})_t=1.24\frac{l}{r_0^3}F_t \qquad \text{(V-13)}$$

For the standard horn, with $\gamma l=2.9$,

$$(\sigma_{\max})_L=8.2\frac{l}{r_0^3}F_L \qquad \text{(V-14)}$$

at the position

$$x_L=0.77l \qquad \text{(V-15)}$$

Hence, the static force applied may be increased by a factor of

$$\frac{F_t}{F_L}=\frac{8.2}{1.24}=6.6 \qquad \text{(V-16)}$$

for the same maximum fiber stress.

The actual permitted increase in clamping force will be greater than the calculated, for the superposed oscillating fiber stress is less at the cross-section of maximum static fiber stress in the inverted exponential coupler than it is in the ordinary exponential coupler.

A toroid tip and appropriate internally sculptured driving coupler were designed, fabricated, and incorporated into an existing 15 kc. roller-seam welding machine. Successful welding of several materials and gages was accomplished in the power range below 700 watts. The system, in this power range, was more efficient than any disc system previously evaluated. However, serious coupler (not toroid) heating was encountered at powers above about 700 watts. Data and observations indicated that the toroid drive tube was presenting excessive "rotational" vibration stiffness to the motion of the toroid at the driving locus around the inside periphery of the toroid. A tube **60** with narrow longitudinal slots **62** was attached between the coupler **36** and the toroid **40** to rotate (e.g. vibrationally) through a greater angle. This resulted in improved energy delivery and welding.

The importance of proper impedance matching between parts of ultrasonic systems (particularly high-power ultrasonic systems) is well known. The lack of theoretical information thereon for the toroid system necessitated

experimentation aimed at accomplishing an approximate impedance match between the weldment and the toroid, and the toroid and the drive coupler. Accordingly, a toroid of reduced cross-section was designed and constructed for the above mentioned tubular driving member: information thus obtained indicated a greatly improved impedance match. This experimental information suggested that the ratio of the toroid cross-section to the drive tube cross-section needed further reduction to accomplish a satisfactory impedance match. However, this could not be done because reducing the overall toroid ring diameter resulted in insufficient operating clearance for practical welding.

Study and calculation indicated that a toroid-like member of roughly triangular cross-section minimizes surface stresses (actually reducing them greatly from values encountered in discs) and also operates to further improve the necessary impedance match. Preliminary welding evaluation of the triangular cross-section toroid showed the estimated improvement over all previous systems. In preliminary investigations, powers up to 1600 watts were handled. Incorporation of the slotted tube **60** in this unit permitted runs at a power level of 2000 watts. The following table contains data wherein the welding results from the toroid systems are compared with those obtained from the 15 kc. 2 kw. disc tip systems.

TYPICAL WELDING MACHINE SETTINGS FOR GOOD WELDS

Aluminum alloy and temper	Gauge, in.	System	Power, watts	Welding rate, ft./min.
1100–H18	0.012	A	1,200	7.5
		E	1,350	9.1
		F	1,200	9.1
	0.020	A	2,200	5.0
		E	1,600	7.5
3003–H18	0.006	A	700	5.0
		B	425	5.0
		C	250	5.0
		D	225	5.0
		E	220	5.0
	0.012	A	1,800	7.5
		E	1,600	7.5
	0.020	A	*2,000	*7.5
		E	2,000	9.1

System:
A—Original standard system with disc tip.
B—Using original 3.7-in. OD torus.
C—Using original 3.7-in. OD torus with slotted adaptor added.
D—3.0-in. OD torus.
E—Toroid-type system with asymmetrical cross-section.
F—Same as for E but with slotted adaptor added.
*Could not be welded.

These data and the graph in FIGURE 8 indicate the superiority of the toroid system. Almost complete relief of the "rotational" stiffness of the tube at its junction with the toroid is feasible.

It is important that proper coupling to the toroid be accomplished to realize the potential of the system. Further, development along these lines will increase the maximum power delivery to the level necessary to effect welds in the thicker sheet materials at higher welding rates.

In designing an inverted exponential coupling member for use in the present invention, the following well known equation may be used:

$$S=S_0e^{-2Tl} \qquad \text{(V-17)}$$

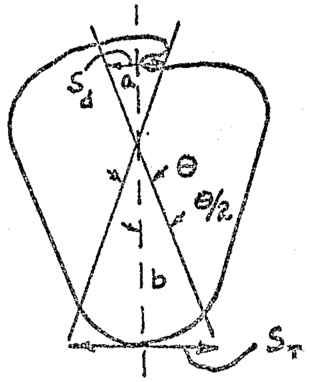
where:

S=original area
S_0=reduced area
T=taper constant
l=length of tapered section.

In designing a toroid element of the present invention the attached theoretical analysis can be used. Subsequent minor frequency adjustment is obtained by removing material at strategic locations. For example, removing material near the periphery increases the frequency while removing material from the sides reduces the frequency.

Schematic representation of a longitudinal resonant element **70** driving an asymmetrical toroidal resonant element **72** through mode-transforming joint **74** is shown in FIGURE 9. The center of mass **76** of this unit lies intermediate the drive point at point **74** and the terminal work end or periphery (P) of this toroidal element at a point a/b of the distance from joint **74** to P.

Relatively simple trigonometric considerations show the influence of this ratio a/b.



If S_d, S_T represent respectively the linear displacement of the drive point at joint **74** and a point on the toroid periphery for an angular rotation θ, then

$$\tfrac{1}{2}S_d = a\theta/2,\ S_d = a\theta$$

and

$$\tfrac{1}{2}S_T = b\theta/2,\ S_T = b\theta$$

from which we see readily that the linear displacement at the periphery exceeds that the drive point precisely as the ratio

$$\frac{1}{b/a} = a/b.$$

Thus, this asymmetrical system shown in FIGURE 9 yields a displacement amplification at P with regard to the displacement at the drive point.

In connection with ascertaining the resonant frequency (and thereby the dimensioning) of a toroid vibrating in revolution about its mean circumference, assume that each section of area A oscillates by revolving about its centroid as a rigid lamina. Let an x–y coordinate system be chosen as shown in FIGURE 12. We assume the section is symmetrical about the y-axis, which passes through the centroid at C. A fiber located at x, y of length $l = 2\pi_y$ changes its length δl when the toroid revolves through a very small angle θ The increase in length is

$$\delta l = 2\pi[c + r\sin(\phi + \theta) - (c + r\sin\phi)] \qquad (1)$$

where r, ϕ are polar coordinates about C, related to x, y by

$$x = r\cos\phi$$
$$y = c + r\sin\phi \qquad (2)$$

where E is modulus of elasticity of the material of the toroid, the fiber stress is

$$T = E\frac{\delta l}{l} = E\frac{r\cos\phi}{c + r\sin\phi}\theta = E\frac{x}{y}\theta \qquad (3)$$

obtained from (1) and (2) on assuming θ small. The total stress over the section vanishes,

$$\int_T dA = E\theta\int\int\frac{x}{y}dxdy = 0 \qquad (4)$$

and the half $x > 0$ is in tension, whereas the other half $x < 0$ is in compression, with the y axis the neutral axis. We neglect all effects connected with Poisson's ratio, which for thick rings will result in some error. The stored elastic strain energy is

$$W = \frac{1}{2}\int\frac{T^2}{E}dv = \frac{1}{2}E\phi^2\int\int\frac{x^2}{y^2}2\pi y\,dxdy$$

or

$$W = \frac{1}{2}E\,\mathcal{J}_1\,\theta^2 \qquad (5)$$

where

$$\mathcal{J}_1 = 2\pi\int\int\frac{x^2}{y}dxdy \qquad (6)$$

is to be evaluated over the section of area

$$A = \int\int dxdy \qquad (7)$$

Next we need an expression for the stored kinetic energy of rotation

$$K = \tfrac{1}{2}I\dot{\theta}^2 \qquad (8)$$

where I is the moment of inertia of the toroid when revolving about its centroidal circumference. Evidently

$$I = \int[x^2 + (y - c)^2]\rho dv$$

or

$$I = \rho\,\mathcal{J}_2 \qquad (9)$$

where

$$\mathcal{J}_2 = 2\pi\int\int[x^2 + (y - c)^2]y\,dxdy \qquad (10)$$

In terms of this definite integral

$$K = \frac{1}{2}\rho\,\mathcal{J}_2\,\dot{\theta}^2 \qquad (11)$$

The total energy, when oscillating, is

$$K + W = \frac{1}{2}\rho\,\mathcal{J}_2\,\dot{\theta}^2 + \frac{1}{2}E\,\mathcal{J}_1\,\theta^2 = \text{constant} \qquad (12)$$

On differentiating (12) with respect to time, we obtain the equation for simple harmonic motion

$$\ddot{\theta} + \frac{E\,\mathcal{J}_1}{\rho\,\mathcal{J}_2}\theta = 0$$

indicating that the resonant frequency is

$$f = \frac{1}{2\pi}\sqrt{\frac{E}{\rho}}\sqrt{\frac{\mathcal{J}_1}{\mathcal{J}_2}} \qquad (13)$$

The integrals

$$\mathcal{J}_1$$

and

$$\mathcal{J}_2$$

must be evaluated numerically for a (symmetrical) section of arbitrary shape. They may be evaluated in closed form for a toroid of circular section.

For this case,

$$\mathcal{J}_1 = 2\pi\int_{-\mathrm{a}}^{+\mathrm{a}}\left[\frac{1}{4}\int_{-\mathrm{a}\cos\phi}^{\mathrm{a}\cos\phi}x^2dx\right]dy$$

$$= \frac{4}{3}\pi\int_{-\mathrm{a}}^{\mathrm{a}}a^3\cos^{\mathrm{a}3}\phi\frac{dy}{y} = \frac{4}{3}\pi\int_{\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{a^4\cos^4\phi}{c + a\sin\phi}d\phi$$

where the change of variable $y = a\sin\phi$, $dy = a\cos\phi d\phi$, on the boundary of the circle, has been made in the last integral. The integration is carried out by expanding

$(1+(a/c)\ \sin\ \phi)^{-1}$ in a series and integrating term by term using the standard definite integral

$$\int_0^{\frac{\pi}{2}} \cos\ {}^n\phi d\phi = \frac{1.3.5(n-1)}{2.4.6n}\ \frac{\pi}{2}$$

One finds that

$$\mathcal{J}_1 = \frac{1}{2}\pi^2\frac{a^4}{c}\left(1+\frac{1}{6}\frac{a^2}{c^2}+\frac{1}{16}\frac{a^4}{c^4}+\frac{1}{32}\frac{a^6}{c^6}+\cdots\right) \tag{14}$$

The other integral is evaluated in similar fashion, with the result that

$$\mathcal{J}_2 = \pi^2 a^4 c \tag{15}$$

Hence, the resonant frequency is

$$f = \frac{1}{2\pi}\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{c}\cdot\sqrt{\frac{E}{\rho}}\cdot\left(1+\frac{1}{12}\frac{a^2}{c^2}+\frac{1}{36}\frac{a^4}{c^3}+\cdots\right) \tag{16}$$

This result, without the infinite series in $(a/c)^2$, is given by Love,[2], p. 453, where he must have assumed the toroid to be so slender that $a/c << 1$.

If the dimensions of the toroid are such that the mean radius C is large compared with the linear dimensions of the section through the solid ring, making the toroid "slender," then the variation of the factor y in the integrals

$$\mathcal{J}_1$$

and

$$\mathcal{J}_2$$

is small and y can be replaced by the mean radius C. In this case

$$\mathcal{J}_1 = \frac{2\pi}{c}\int x^2 dA = \frac{2\pi}{c} I_x \tag{17}$$

and

$$\mathcal{J}_2 = 2\pi c\int r^2 dA = 2\pi c I_r \tag{18}$$

where I_x is the moment of area about the y or neutral axis, and I_r is the polar moment of area about the centroid. The frequency is then given by

$$f = \frac{1}{2\pi}\frac{1}{c}\sqrt{\frac{E}{P}\frac{I_x}{I_r}} \tag{19}$$

For a circular section of radius a, $I_x = \frac{1}{2}a^4$ and $I_r = \frac{1}{4}a^4$ so that one obtains Equation 16 without the infinite series.

For a thick toroid in which the variation in y is appreciable, the integral

$$\mathcal{J}_1$$

can be evaluated most simply as follows:
First integrate x to obtain

$$\mathcal{J}_1 = \frac{4\pi}{3}\int_{y_{min}}^{y_{max}} \frac{1}{y}[x_B(y)]^3 dy \tag{20}$$

where $x_B(y)$ is the value of x on the boundary of the section. Now change the variable y to $Z = \ln y$ so that Equation 20 becomes

$$\mathcal{J}_1 = \frac{4\pi}{3}\int_{Z_{min}}^{Z_{max}} [x_B(Z)]^3 dZ \tag{21}$$

This integral may be evaluated graphically be plotting $[x_B(Z)]^3$ against Z, and taking the area under the curve between the limits Z_{min} to Z_{max}.

The integral

$$\mathcal{J}_2$$

can be evaluated in somewhat similar fashion by splitting it into the sum of two integrals and evaluating each by integrating x first, holding y constant, then carrying out a graphical integration for the integration over y, after making a suitable change in variable.

As shown more clearly in FIGURE 13, the toroid 72 may be connected to exponential type coupler 80 by means of a plurality of discrete ribbons 82. The small end of the coupler 80 is provided with a notch 84 within which one end of the ribbons 82 is secured as by brazing. The other end of the ribbons 82 is secured to the toroid 72 at the inner peripheral surface thereof.

While only four ribbons are illustrated in FIGURE 13, a greater or lesser number may be provided. The number of ribbons provided depends upon the power requirements and the degree of acoustic impedance match desired between the ends of the ribbons and the resonant toroid 72.

While the ribbons may be of different linear lengths, this is compensated by the bend radius of the ribbons so that each ribbon is exactly $nx/2$ wavelengths long, where n is an integer. For optimum operation, the length of each ribbon must be an integral number of one-half wavelengths in the material at the system frequency so that a loop is obtained at both ends of the ribbons. A more detailed explanation of the design structure of the ribbons is set forth in copending application Serial Number 120,233 filed on June 28, 1961 and entitled Apparatus and Methyl for Introducing High Levels of Vibratory Energy to a Work Area.

The present invention may be embodied in other specific forms without departing from the spirit or essential attributes thereof and, accordingly reference should be made to the appended claims, rather than to the foregoing specification as indicating the scope of the invention.

I claim:

1. A non-fusion method for welding metal members togther which comprises placing to-be-welded faces of the metal members in intimate contact, applying a force to the metal members in a direction and of a magnitude sufficient to hold the contacting to-be-welded faces of the metal members in intimate contact at the intended weld zone and to couple mechanical vibratory energy at a frequency of between 59 and 300,000 cycles per second into said weld zone, causing a toroid to vibrate in torsion by introducing linear vibratory mechanical energy to the inner periphery of the toroid resonant at said frequency, and introducing to one of the to-be-welded metal members adjacent the weld zone torsional vibratory energy from the outer periphery of said toroid, with the vibratory energy introduced to said one metal member being at a sufficient energy level to weld the metal members together.

2. A method in accordance with claim 1 including the step of amplifying the amplitude of vibratory energy in the toroid so that its magnitude at the outer periphery of the toroid is greater than that introduced to the inner periphery of the toroid.

3. A method of delivering vibratory energy comprising the steps of axially vibrating a coupler at a frequency of between 59 and 300,000 cycles per second, torsionally vibrating a toroid by using the axial vibrations in said coupler, said last-mentioned step including coupling one end of said coupler to said toroid in a manner so that the center of mass of said toroid is intermediate its periphery and the area of said toroid to which said coupler is coupled.

4. A method in accordance with claim 3 wherein said periphery of said toroid is the outer periphery thereof, the axial vibrations are delivered to said toroid at a point adjacent its inner periphery.

5. A method in accordance with claim 4 wherein said periphery of said toroid is the outer periphery thereof, said toroid is generally triangular in transverse cross-sec-

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[2] A. E. H. Love, A Treatise on the Mathematical Theory of Elasticity, 4th revised edition, New York, Dover Publications, 1944.

tion with the base of the triangular cross-section being the inner periphery of said toroid, and amplifying the amplitude of vibratory energy capable of being delivered by the outer periphery of said toroid.

6. A method in accordance with claim 4 including relieving rotational stresses at the juncture intermediate the inner periphery of said toroid and said one end of said coupler.

7. Apparatus for non-fusion welding together the contacting surfaces of a plurality of metal members comprising a coupler bar, means for generating vibratory energy at a frequency of between 59 and 300,000 cycles per second connected axially to a first end of said coupler bar, a toroid resonant at said frequency coupled to a second end of said coupler bar at an area spaced from the center of mass of said toroid as measured in a radial direction on said toroid, means for applying a force to the metal members in a direction and of a magnitude sufficient to hold contacting to-be-welded faces of the metal members in intimate contact at the intended weld zone and to couple mechanical vibratory energy into said zone from said toroid, whereby said toroid may present vibratory energy to the metal members at an energy level sufficient to weld the metal members together.

8. Apparatus in accordance with claim 7 wherein said coupler bar is annular in transverse cross section adjacent its second end.

9. Apparatus in accordance with claim 8 wherein said second end of said coupler bar being fixedly and continuously secured to the entire inner periphery of said toroid.

10. Apparatus in accordance with claim 7 wherein said toroid is generally triangular in transverse cross section, with the base of said triangle being on the inner periphery of said toroid, and said second end of said coupler bar being fixedly secured to said toroid adjacent the inner periphery of said toroid, with the distance between center of mass of said toroid and its outer periphery being greater than the distance between said center of mass and the inner periphery of said toroid.

11. Apparatus in accordance with claim 7 including a slotted tube connected to said second end of said coupler bar, said tube being fixedly secured to the inner periphery of said toroid.

12. Apparatus comprising a means for generating vibratory energy at a frequency of between 59 and 300,000 cycles per second, a coupler bar axially joined at one end to said means, a toroid resonant at said frequency, and means mounting said toroid on the other end of said coupler bar so that the outer periphery of said toroid delivers torsional vibratory energy for performing useful work, the center of mass of said toroid being closer to the outer periphery of said coupler bar than said outer periphery of said toroid.

13. Apparatus in accordance with claim 12 wherein said mounting means includes a slotted tube, one end of said tube being axially joined to the other end of said coupler bar, the other end of said tube being fixedly secured to the inner periphery of said toroid, said coupler bar other end being internally sculptured adjacent said tube so as to have an inverted exponential taper.

14. Apparatus in accordance with claim 12 wherein said toroid is generally triangular in transverse cross section, the base of said triangle being adjacent the inner periphery of said toroid, and said mounting means fixedly securing said coupler bar to said toroid adjacent the inner periphery of said toroid.

15. Apparatus in accordance with claim 12 wherein said mounting means includes a lip on the inner periphery of said toroid.

References Cited by the Examiner

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